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An LMI Method to Robust Iterative Learning Fault-tolerant Guaranteed Cost Control for Batch Processes*

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Abstract Based on an equivalent two-dimensional Fornasini-Marchsini model for a batch process in industry, a closed-loop robust iterative learning fault-tolerant guaranteed cost control scheme is proposed for batch processes with actuator failures. This paper introduces relevant concepts of the fault-tolerant guaranteed cost control and formulates the robust iterative learning reliable guaranteed cost controller (ILRGCC). A significant advantage is that the proposed ILRGCC design method can be used for on-line optimization against batch-to-batch process uncertainties to realize robust tracking of set-point trajectory in time and batch-to-batch sequences. For the convenience of implementation, only measured output errors of current and previous cycles are used to design a synthetic controller for iterative learning control, consisting of dynamic output feedback plus feed-forward control. The proposed controller can not only guarantee the closed-loop convergency along time and cycle sequences but also satisfy the H_∞ performance level and a cost function with upper bounds for all admissible uncertainties and any actuator failures. Sufficient conditions for the controller solution are derived in terms of linear matrix inequalities (LMIs), and design procedures, which formulate a convex optimization problem with LMI constraints, are presented. An example of injection molding is given to illustrate the effectiveness and advantages of the ILRGCC design approach.
Keywords two-dimensional Fornasini-Marchsini model, batch process, iterative learning control, linear matrix inequality, fault-tolerant guaranteed cost control

1 INTRODUCTION

Batch processing technologies have received widespread concerns in the past 10 years since batch processes are preferred for manufacturing low-volume and high-value products [1]. Though the study on batch process control can be dated back to 1930s [2], the process optimization and control lags far behind the development for continuous production processes. To guarantee the quality and quality consistency of batch processes, advanced control is critical importance.

The high productivity demand pushes chemical plants to operate under challenging conditions, which of course leads to the possibility of system failures. If a failure is not controlled promptly with a proper corrective action, it will degrade the process performance, and in serious cases, result in safety problems for the plant and personnel. Fault detection and diagnosis can detect and estimate the faults [3, 4], whereas fault-tolerant control (FTC) is capable of maintaining the performance of closed-loop systems at an acceptable level in the presence of faults. Among those FTC methods, reliable control is popular [5]. The study on reliable control has received considerable attention for continuous processes because of the growing demands on reliability. However, only limited results on FTC for batch processes are available [6–8].

In harmony with the repetitive nature of batch process, iterative learning control (ILC) has been used widely in recent years for industrial and chemical batch processes to realize perfect tracking and control optimization [9–13]. However, in practice, many batch processes are slowly time-varying from batch to batch. ILC methods cannot hold robust stability for processes varying from batch to batch [14]. Recently, Shi *et al*. proposed a more general design framework for ILC of batch process, *i.e*., the feedback integrated with ILC method [15–20]. The results are obtained in the normal case. For faulty cases, there exist few results. Wang *et al*. [6] developed a 2D iterative learning reliable control (ILRC) for batch processes with actuator failures. The fault-tolerant control scheme for batch processes with sensor faults was also studied [7, 8].

In the robust controller design, we are concerned with not only the robust stability of an uncertain closed-loop system, but also its robust performance, which is more important when controlling a system dependent on uncertain parameters. That leads to the so-called guaranteed cost control approach, first introduced by Chang and Peng [21]. Such problem for 1D and 2D discrete uncertain systems has received considerable attention and robust controller design methods have been established [22–24]. Although the

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results can be extended to robust iterative learning guaranteed cost controller design, no results on such issue are available.

In this paper, based on the 2D system theory, we design a robust iterative learning reliable guaranteed cost controller (ILRGCC) for batch processes with actuator failures. Different from previous investigations [7, 8], this ILRGCC intends to preserve not only the H_∞ performance, but also the least guaranteed cost function with upper bounds for all admissible uncertainties and any actuator failures. Since the system states cannot often measured in practical applications, the designed synthetic ILC controller consists of dynamic output feedback plus feed-forward control. Sufficient conditions for the proposed fault tolerance guaranteed cost control are expressed as linear matrix inequalities (LMIs) and design procedures are presented in terms of a convex optimization problem with LMI constraints. Finally, the feasibility and effectiveness of the proposed method are demonstrated with injection velocity control.

2 PROBLEM DESCRIPTION

Process $\sum_{\mathrm{P}}$, which is referred to a process repetitively performing a task over a certain period of time called a cycle, can be described by the following discrete-time model with uncertain parameter perturbations

$$\sum_{\mathrm{P}}: \begin{cases} \boldsymbol{x}(t+1,k)=[\boldsymbol{A}+\boldsymbol{\varDelta}_a(t,k)]\boldsymbol{x}(t,k)+\boldsymbol{Bu}(t,k) \\ \boldsymbol{y}(t,k)=\boldsymbol{Cx}(t,k) \end{cases}$$
$$\text{for} \quad \boldsymbol{x}(0,k)=\boldsymbol{x}_{0,k};\quad t=0,1,2,\cdots,T;\ k=1,2,\cdots \tag{1}$$

where k and t are cycle and time index, respectively, and $\boldsymbol{x}_{0,k}$ is the time-wise initial state of the kth cycle. $\boldsymbol{x}(t,k)\in\boldsymbol{R}^n$, $\boldsymbol{y}(t,k)\in\boldsymbol{R}^l$ and $\boldsymbol{u}(t,k)=[u_1(t,k), u_2(t,k),\cdots,u_m(t,k)]\in R^m$ are, respectively, the state, output, and input of the process at time t in the kth cycle. Meanwhile, $\boldsymbol{R}^n$ represents Euclidean n space, with the norm denoted by $\|\bullet\|$.{A, B, C} are constant matrices of appropriate dimensions, and $\varDelta_{\mathrm{a}}(t,k)$ is a perturbation at time t in the kth cycle and can be specified as $\varDelta_{\mathrm{a}}(t,k)=\boldsymbol{E}\varDelta(t,k)\boldsymbol{F}$ with $\varDelta^{\mathrm{T}}(t,k)\varDelta(t,k)\leqslant\boldsymbol{I}$, $0\leqslant t\leqslant T$; $k=1,2,\cdots$, where {E, F} are known constant matrices. For any two sequential cycles, $\delta[\varDelta_{\mathrm{a}}(t,k)]=\varDelta_{\mathrm{a}}(t,k)-\varDelta_{\mathrm{a}}(t,k-1)$ is the cycle-to-cycle parameter perturbation. Generally, $\varDelta(t,k)$ is represented as a function of time t and cycle k. If $\varDelta(t,k)$ depends on time t only, it has $\delta[\varDelta_{\mathrm{a}}(t,k)]=0$, which is called a repeatable perturbation; otherwise, $\delta[\varDelta_{\mathrm{a}}(t,k)]\neq 0$, the perturbation is non-repeatable.

For control input $u_i(t,k)$ ($i=1,2,\cdots,m$), let $u_i^{\mathrm{F}}(t,k)$ denote the signal from the failed actuator. The failure model can be represented as

$$u_i^{\mathrm{F}}(t,k)=\alpha_i u_i(t,k),\quad (\text{for } i=1,2,\cdots,m) \tag{2}$$

where

$$0\leqslant\underline{\alpha}_i\leqslant\alpha_i\leqslant\bar{\alpha}_i,\quad (\text{for } i=1,2,\cdots,m) \tag{3}$$

The terms $\underline{\alpha}_i$ ($\underline{\alpha}_i\leqslant 1$) and $\bar{\alpha}_i$ ($\bar{\alpha}_i\geqslant 1$) are known scalars.

The failure model expressed by Eq. (2) is widely adopted. The parameter α_i is unknown but is assumed to vary within a known range, which can be described by four forms: $\underline{\alpha}_i=\bar{\alpha}_i$ corresponding to the normal case $u_i^{\mathrm{F}}(t,k)=u_i(t,k)$; $\alpha_i>0$ representing a partial failure case, *i.e.*, partial degradation of the actuator; $\alpha_i=0$ covering the outage case and the stuck fault making the output of an actuator at a constant value. When encountering the last two failures, the system will no longer have the controllability and trouble-shooting is needed along with the use of monitoring tools, which is out of the range of this work. Therefore, we only consider $\alpha_i>0$ here. Denote

$$\boldsymbol{u}^{\mathrm{F}}=\left[u_1^{\mathrm{F}},u_2^{\mathrm{F}},\cdots,u_m^{\mathrm{F}}\right]^{\mathrm{T}} \tag{4a}$$

$$\bar{\boldsymbol{\alpha}}=\mathrm{diag}\left[\bar{\alpha}_1,\bar{\alpha}_2,\cdots,\bar{\alpha}_m\right] \tag{4b}$$

$$\underline{\boldsymbol{\alpha}}=\mathrm{diag}\left[\underline{\alpha}_1,\underline{\alpha}_2,\cdots,\underline{\alpha}_m\right] \tag{4c}$$

$$\boldsymbol{\alpha}=\mathrm{diag}\left[\alpha_1,\alpha_2,\cdots,\alpha_m\right] \tag{4d}$$

Meanwhile, define the following notations:

$$\boldsymbol{\beta}=\mathrm{diag}\left[\beta_1,\beta_2,\cdots,\beta_m\right] \tag{5a}$$

$$\boldsymbol{\beta}_0=\mathrm{diag}\left[\beta_{10},\beta_{20},\cdots,\beta_{m0}\right] \tag{5b}$$

with

$$\beta_i=\frac{\bar{\alpha}_i+\underline{\alpha}_i}{2},\quad (\text{for } i=1,2,\cdots,m) \tag{5c}$$

From Eqs. (4) and (5), for some unknown matrix $\boldsymbol{\alpha}_0$, $\boldsymbol{\alpha}$ can be expressed as

$$\boldsymbol{\alpha}=(\boldsymbol{I}+\boldsymbol{\alpha}_0)\boldsymbol{\beta} \tag{6}$$

with

$$|\boldsymbol{\alpha}_0|\leqslant\beta_0\leqslant I \tag{7}$$

where

$\boldsymbol{\alpha}_0\triangleq\mathrm{diag}\left[\alpha_{01},\alpha_{02},\cdots,\varepsilon_{0m}\right]$, $|\boldsymbol{\alpha}_0|\triangleq\mathrm{diag}[|\alpha_{01}|,|\alpha_{02}|,\cdots,|\alpha_{0m}|]$, and $|\bullet|$ denotes absolute value of "•".

A batch process with actuator failures is described by

$$\sum_{\mathrm{P-F}}: \begin{cases} \boldsymbol{x}(t+1,k)=[\boldsymbol{A}+\boldsymbol{\varDelta}_{\mathrm{a}}(t,k)]\boldsymbol{x}(t,k)+\boldsymbol{B\alpha u}(t,k) \\ \boldsymbol{y}(t,k)=\boldsymbol{Cx}(t,k) \\ \boldsymbol{x}(t,k)=\boldsymbol{x}_{0,k};\ t=0,1,2,\cdots,T;\ k=1,2,\cdots \end{cases} \tag{8}$$

Based on the batch process with actuator failures described by Eq. (8), the motive of this work can be stated as deriving the control law $\boldsymbol{u}(t, k)$, which can maintain the robust stability of system (8) and minimize the performance cost at the same time.

3 RESULTS

3.1 Equivalent 2D model representation

For process $\sum_{\mathrm{P}}$ described by system (1), define an ILC law in the form of

$$\sum\nolimits_{\mathrm{ilc}}: \ \boldsymbol{u}(t,k)=\boldsymbol{u}(t,k-1)+\boldsymbol{r}(t,k)$$
$$[\text{for } \ \boldsymbol{u}(t,0)=0, \quad t=0,1,2,\cdots,T\ ] \tag{9}$$

where $\boldsymbol{u}(t,0)$ is the initial value of iteration and $\boldsymbol{r}(t,k)\in R^m$ is called the updating law of the ILC to be determined. The objective of the ILC design is to establish a procedure for the design of a reliable guaranteed cost controller, Eq. (9) or equivalent updating law $r(t,k)$, so that $\boldsymbol{y}(t,k)$ tracks the given setpoint trajectory $y_{\mathrm{d}}(t)$ and the closed-loop system to be represented preserves an adequate control performance.

Design the output tracking error in the current cycle as

$$\boldsymbol{e}(t,k)=\boldsymbol{y}(t,k)-y_{\mathrm{d}}(t) \tag{10a}$$

Meanwhile, define a batchwise direction function of error as

$$\delta_k[f(t,k)]=f(t,k)-f(t,k-1) \tag{10b}$$

where $f(t,k)$ may be represented state variable. With system (1) along with the definition Eq. (10), we have

$$\delta_k[\boldsymbol{x}(t+1,k)]=[\boldsymbol{A}+\boldsymbol{\Delta}_{\mathrm{a}}(t,k)]\delta_k[\boldsymbol{x}(t,k)]+\boldsymbol{B\alpha r}(t,k)+\boldsymbol{\omega}(t,k) \tag{11a}$$

$$\begin{aligned}\boldsymbol{e}(t+1,k)&=\boldsymbol{y}(t+1,k)-y_{\mathrm{d}}(t)\\&=\boldsymbol{e}(t+1,k-1)+\boldsymbol{C}\delta_k[\boldsymbol{x}(t+1,k)]\\&=\boldsymbol{e}(t+1,k-1)+\boldsymbol{C}[\boldsymbol{A}+\boldsymbol{\Delta}_{\mathrm{a}}(t,k)]\delta_k[\boldsymbol{x}(t,k)]+\\&\quad\boldsymbol{CB\alpha r}(t,k)+\boldsymbol{C\omega}(t,k)\end{aligned} \tag{11b}$$

where $\boldsymbol{\omega}(t,k)$ is the perturbation and can be expressed by

$$\boldsymbol{\omega}(t,k)=\delta_k[\boldsymbol{\Delta}_{\mathrm{a}}(t,k)]\boldsymbol{x}(t,k-1) \tag{11c}$$

Obviously, for repeatable perturbations, $\omega(t,k)=0$; for a non-repeatable disturbance, $\omega(t,k)\neq 0$. Thereby, an equivalent 2D error fault model $\sum_{\mathrm{2D-ep-F}}$ description for the above batch process can be rewritten as

$$\sum\nolimits_{\mathrm{2D-ep-F}}:\begin{cases}\boldsymbol{x}_1(t+1,k)=[\boldsymbol{A}_1+\boldsymbol{\Delta}_{\mathrm{A}_1}(t,k)]\boldsymbol{x}_1(t,k)+\boldsymbol{A}_2\boldsymbol{x}_1(t+1,k-1)+\bar{\boldsymbol{B}}\boldsymbol{\alpha r}(t,k)+\boldsymbol{G\omega}(t,k)\\ \qquad\qquad\ \ =\boldsymbol{A}_1(t,k)\boldsymbol{x}_1(t,k)+\boldsymbol{A}_2\boldsymbol{x}_1(t+1,k-1)+\bar{\boldsymbol{B}}\boldsymbol{\alpha r}(t,k)+\boldsymbol{G\omega}(t,k)\\ \boldsymbol{y}(t,k)=\begin{bmatrix}\boldsymbol{e}(t,k-1)\\ \boldsymbol{e}(t,k)\end{bmatrix}=\bar{\boldsymbol{C}}\boldsymbol{x}_1(t,k)\\ \boldsymbol{Z}(t,k)\triangleq\boldsymbol{e}(t,k)=\boldsymbol{H}\boldsymbol{x}_1(t,k)\end{cases} \tag{12}$$

where $\boldsymbol{Z}(t,R)\in\boldsymbol{R}^{\mathrm{P}}$ is controlled output, $\boldsymbol{x}_1(t,k)=\begin{bmatrix}\delta_k[\boldsymbol{x}(t,k)]\\ \boldsymbol{e}(t,k)\end{bmatrix}$, $\boldsymbol{A}_1=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{CA} & \boldsymbol{0}\end{bmatrix}$, $\boldsymbol{A}_2=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$, $\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{CB}\end{bmatrix}$, $\boldsymbol{\Delta}_{\mathrm{A}_1}(t,k)=\bar{\boldsymbol{E}}\boldsymbol{\Delta}(t,k)\bar{\boldsymbol{F}}$, $\bar{\boldsymbol{E}}=\begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{CE}\end{bmatrix}$, $\bar{\boldsymbol{F}}=\begin{bmatrix}\boldsymbol{F} & \boldsymbol{0}\end{bmatrix}$, $\boldsymbol{G}=\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{C}\end{bmatrix}$, $\boldsymbol{H}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$, and $\bar{\boldsymbol{C}}=\begin{bmatrix}-\boldsymbol{C} & \boldsymbol{I}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$.

The 2D error fault model $\sum_{\mathrm{2D-ep-F}}$, a typical two-dimensional Fornasini-Marchsini (2D-FM) model with uncertain perturbations, equivalently represents the dynamical behavior of the tracking error of system (1). It is called the equivalent 2D tracking error model of system (1). Therefore, the design of the updating law $r(t,k)$ for system (1) is clearly equivalent to the design of a reliable guaranteed cost control law for the equivalent 2D tracking error model $\sum_{\mathrm{2D-ep-F}}$. Accordingly, for the system $\sum_{\mathrm{2D-ep-F}}$, we assume that it has a finite set of initial conditions, *i.e.*, there exist two positive integers t and k such that

$$\boldsymbol{x}_1(t,0)=0, \quad t\geqslant r_1; \quad \boldsymbol{x}_1(0,k)=0, \quad k\geqslant r_2 \tag{13a}$$

where $r_1<\infty$ and $r_2<\infty$ are positive integers. The initial boundary conditions are arbitrary, but belong to the set

$$\begin{aligned}S=\{&\boldsymbol{x}_1(t,0),\boldsymbol{x}_1(0,k)\in\boldsymbol{R}^n:\boldsymbol{x}_1(t,0)\\&=\boldsymbol{Mv}_1,\ \boldsymbol{x}_1(0,k)=\boldsymbol{Mv}_2,\boldsymbol{v}_t^{\mathrm{T}}\boldsymbol{v}_t\leqslant I,\ (t=1,2)\}\end{aligned} \tag{13b}$$

where $\boldsymbol{M}$ is a given matrix, $\boldsymbol{x}_1(t,0)$, $\boldsymbol{x}_1(0,k)\in\boldsymbol{R}^n$ are elements in the set and expressed as the form behind the colon.

The state feedback control is usually impracticable, so we introduce the following dynamic 2D output feedback controller which is represented by model $\sum_{\mathrm{2D-ep-F}}^{\mathrm{c}}$ for the 2D-FM system $\sum_{\mathrm{2D-ep-F}}$ in the form of

$$\Sigma_{\text{2D-ep-F}}^{\text{c}}:\begin{cases}\boldsymbol{x}_{\text{c}}(t+1,k)=\boldsymbol{A}_{\text{c}1}\boldsymbol{x}_{\text{c}}(t,k)+\boldsymbol{A}_{\text{c}2}\boldsymbol{x}_{\text{c}}(t+1,k-1)+\boldsymbol{B}_{\text{c}1}\boldsymbol{y}(t,k)+\boldsymbol{B}_{\text{c}2}\text{y}(t+1,k-1)\\ \boldsymbol{r}(t,k)=\boldsymbol{C}_{\text{c}1}\boldsymbol{x}_{\text{c}}(t,k)+\boldsymbol{C}_{\text{c}2}\boldsymbol{x}_{\text{c}}(t+1,k-1)+\boldsymbol{D}_{\text{c}1}\boldsymbol{y}(t,k)+\boldsymbol{D}_{\text{c}2}\boldsymbol{y}(t+1,k-1)\end{cases}\tag{14}$$

where $\boldsymbol{x}_{\text{c}}(t,k)\in\boldsymbol{R}^n$ is the internal state of the controller and $\{\boldsymbol{A}_{\text{c}i},\boldsymbol{B}_{\text{c}i},\boldsymbol{C}_{\text{c}i},\boldsymbol{D}_{\text{c}i}\}_{i=1,2}$ are controller parameters to be determined. The 2D closed-loop error fault system $\Sigma_{\text{2D-ep-F}}$ obtained (Fig. 1) by substituting controller (14) into system (12) is represented as

$$\Sigma_{\text{2D-ep-F-o}}:$$

$$\begin{cases}\boldsymbol{x}_{\text{r}}(t+1,k)=[\tilde{\boldsymbol{A}}_1+\tilde{\boldsymbol{\varDelta}}_{\text{a}}(t,k)]\boldsymbol{x}_r(t,k)+\\ \qquad\qquad\tilde{\boldsymbol{A}}_2\boldsymbol{x}_r(t+1,k-1)+\tilde{\boldsymbol{G}}\boldsymbol{\omega}(t,k)\\ \boldsymbol{y}(t,k)\triangleq\tilde{\boldsymbol{C}}\boldsymbol{x}_{\text{r}}(t,k)\\ \boldsymbol{z}(t,k)\triangleq\boldsymbol{e}(t,k)=\tilde{\boldsymbol{H}}\boldsymbol{x}_{\text{r}}(t,k)\end{cases}\tag{15}$$

where $\boldsymbol{x}_{\text{r}}(t+1,k)=\begin{bmatrix}\boldsymbol{x}_1(t+1,k)\\ \boldsymbol{x}_{\text{c}}(t+1,k)\end{bmatrix}$ is the state of the 2D closed-loop system, $\tilde{\boldsymbol{A}}_i=\begin{bmatrix}\boldsymbol{A}_i+\bar{\boldsymbol{B}}\boldsymbol{\alpha}\boldsymbol{D}_{\text{c}i}\bar{\boldsymbol{C}} & \bar{\boldsymbol{B}}\boldsymbol{\alpha}\boldsymbol{C}_{\text{c}i}\\ \boldsymbol{B}_{\text{c}i}\bar{\boldsymbol{C}} & \boldsymbol{A}_{\text{c}i}\end{bmatrix}$, $(i=1,2)$, $\tilde{\boldsymbol{G}}=\begin{bmatrix}\boldsymbol{G}\\ \boldsymbol{0}\end{bmatrix}$, $\tilde{\boldsymbol{C}}=\begin{bmatrix}\bar{\boldsymbol{C}} & \boldsymbol{0}\end{bmatrix}$, $\tilde{\boldsymbol{H}}=\begin{bmatrix}\boldsymbol{H} & \boldsymbol{0}\end{bmatrix}$, and $\tilde{\varDelta}_{\text{a}}(t)=\begin{bmatrix}\varDelta_{A_1}(t,k) & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}=\begin{bmatrix}\bar{\boldsymbol{E}}\\ \boldsymbol{0}\end{bmatrix}\varDelta(t,k)\begin{bmatrix}\bar{F} & 0\end{bmatrix}=\tilde{\boldsymbol{E}}\varDelta(t,k)\tilde{\boldsymbol{F}}$.

Accordingly, the boundary conditions of the 2D system $\Sigma_{\text{2D-ep-F-o}}$ are assumed as

$$\boldsymbol{x}_{\text{r}}(t,0)=\begin{bmatrix}\boldsymbol{x}_1(t,0)\\ \boldsymbol{0}\end{bmatrix},\quad \boldsymbol{x}_{\text{r}}(0,k)=\begin{bmatrix}\boldsymbol{x}_1(0,k)\\ \boldsymbol{0}\end{bmatrix}\tag{16a}$$

and the initial boundary conditions belong to the set

$$S=\{\boldsymbol{x}_{\text{r}}(t,0),\boldsymbol{x}_{\text{r}}(0,k)\in\boldsymbol{R}^n:\boldsymbol{x}_{\text{r}}(t,0)=\bar{\boldsymbol{M}}\bar{\boldsymbol{v}}_1,\ \boldsymbol{x}_{\text{r}}(0,k)=\bar{\boldsymbol{M}}\bar{\boldsymbol{v}}_2,\bar{\boldsymbol{v}}_t^{\text{T}}\bar{\boldsymbol{v}}_t\leqslant\boldsymbol{I},\ (t=1,2)\}\tag{16b}$$

where $\bar{M}=\text{diag}[\boldsymbol{M}\ \ \boldsymbol{0}]$, $\boldsymbol{x}_{\text{r}}(t,0)$, $\boldsymbol{x}_{\text{r}}(0,k)\in\boldsymbol{R}^n$ are elements in the set and expressed as the form behind the colon.

Denoting $\bar{x}_{\text{r}}=\sup\{\|x_{\text{r}}(t,k)\|:t+k=N,\ t,k\geqslant1\}$, we first give the definition of asymptotic stability for system (12).

Definition 1 [25] The uncertain 2D system (12) is asymptotically stable if $\lim_{N\to\infty}\bar{x}_{\text{r}}=0$ with zero input $r(t,k)=0$ and $\omega(t,k)=0$.

Associated with 2D system (15) is the following cost function

$$\begin{aligned}J&=\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\left[\boldsymbol{x}_{\text{r}}^{\text{T}}(t,k)\boldsymbol{U}_1\boldsymbol{x}_{\text{r}}(t,k)+\boldsymbol{x}_{\text{r}}^{\text{T}}(t+1,k-1)\boldsymbol{U}_2\boldsymbol{x}_{\text{r}}(t+1,k-1)+\boldsymbol{r}^{\text{T}}(t,k)\boldsymbol{U}_3\boldsymbol{r}(t,k)\right]\\&=\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\boldsymbol{\varphi}^{\text{T}}(t,k)\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\text{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\text{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\text{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\text{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}\boldsymbol{\varphi}(t,k)\end{aligned}\tag{17}$$

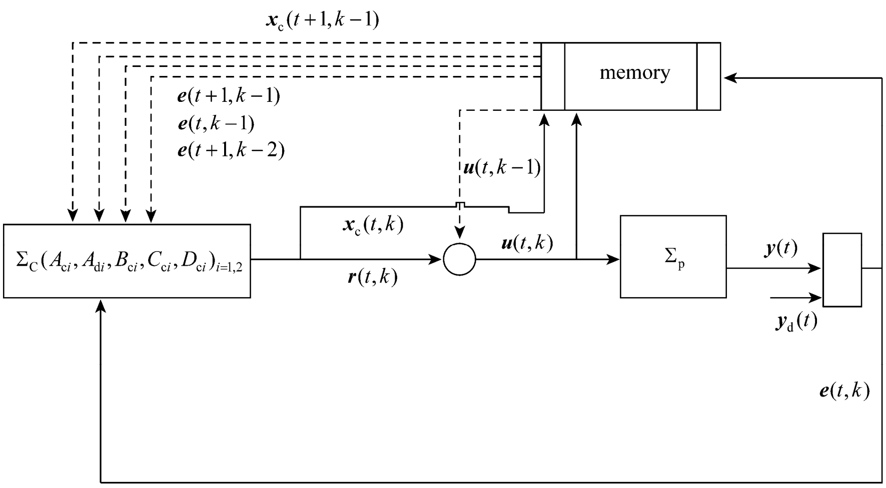

Figure 1 Schematic diagram of the structure of a closed-loop system

where $\boldsymbol{\varphi}^{\mathrm{T}}(t,k)=\left[\boldsymbol{x}_{\mathrm{r}}^{\mathrm{T}}(t,k) \quad x_{\mathrm{r}}^{\mathrm{T}}(t+1,k-1)\right]$, $\boldsymbol{U}_1>0$, $\boldsymbol{U}_2>0$, $\boldsymbol{U}_3>0$, and $\boldsymbol{K}_i=\left[\boldsymbol{D}_{\mathrm{c}i}\bar{\boldsymbol{C}} \quad \boldsymbol{C}_{\mathrm{c}i}\right]$.

Some definitions are introduced to establish a procedure for the design of updating law $\boldsymbol{r}(t,h)$, which guarantees the closed-loop system robust stable and preserving an adequate control performance.

Definition 2 For any bounded boundary conditions satisfying Eq. (16b), all admissible uncertainties and any admissible actuator failures, if there exists a controller $\boldsymbol{r}^*(t,k)$ and some specified constant J^* such that the state of the resulting closed-loop system (15) with $\boldsymbol{\omega}(t,k)=0$ satisfies $\lim_{N\to\infty}\bar{x}_{\mathrm{r}}=0$ and its cost function Eq. (17) satisfies $J\leqslant J^*$, then J^* is said to be a fault-tolerant guaranteed cost, $\boldsymbol{r}^*(t,k)$ is said to be a fault-tolerant guaranteed cost control law for the uncertain 2D system (12), and the closed-loop system $\sum_{\text{2D-ep-F-o}}$ is called a 2D-fault-tolerant guaranteed cost control system.

Definition 3 Control law $\boldsymbol{r}^*(t,k)$ is a robust H_∞ fault-tolerant guaranteed cost control law for the uncertain 2D system (12), if the following conditions hold and there exists a scalar $\gamma>0$, for all admissible parameter uncertainties and any admissible actuator failures,

(1) The resulting closed-loop system (15) with $\boldsymbol{\omega}(t,k)=0$ is asymptotically stable;

(2) With the zero initial condition, the controlled output $\boldsymbol{z}(t,k)$ satisfies

$$\|\boldsymbol{z}\|_{\text{2D-2e}}\leqslant\gamma\|\boldsymbol{\omega}\|_{\text{2D-2e}}$$

where for a 2D signal $\boldsymbol{\omega}(t,k)$ and any integers N_1, $N_2>0$, if $\|\boldsymbol{\omega}(\bullet,\bullet)\|_{\text{2D-2e}}=\sqrt{\sum_{t=0}^{N_1}\sum_{k=0}^{N_2}\|\boldsymbol{\omega}(\bullet,\bullet)\|^2}<\infty$, then $\boldsymbol{\omega}(t,k)$ is said to be in $l_{\text{2D-2e}}$ space, as denoted by $\boldsymbol{\omega}(\bullet,\bullet)\in l_{\text{2D-2e}}$.

(3) In the case of $\omega(t,k)=0$, the cost function for the resulting closed-loop system (15) satisfies $J\leqslant J^*$.

Lemma 1 [18] The 2D closed-loop system $\sum_{\text{2D-ep-F-o}}$ is 2D-fault-tolerant guaranteed cost control if there is a function $V(\bullet)$ that satisfies the following conditions:

(1) $V(x)\geqslant 0$ for $x\in\boldsymbol{R}^n$, and $V(x)=0\leftrightarrow x=0$;

(2) $V(x)\to\infty$ as $\|x\|\to\infty$;

(3) For any boundary conditions, any admissible actuator failures satisfying Eq. (3) and $\forall T_0>0$, $K_0>0$, $i>0$

$$\sum_{\substack{t+k=T_0+K_0+i+1\\ T_0\leqslant t\leqslant T_0+i\\ K_0\leqslant k\leqslant K_0+i}} V[\boldsymbol{x}(t,k)]<\sum_{\substack{t+k=T_0+K_0+i\\ T_0\leqslant t\leqslant T_0+i\\ K_0\leqslant k\leqslant K_0+i}} V[\boldsymbol{x}(t,k)]$$

3.2 Reliable guaranteed cost controller design and system structure

In this section, we will design a reliable updating law $\boldsymbol{r}(t,k)$ such that the resulting closed-loop system (15) is 2D-fault-tolerant guaranteed cost control and the cost function of closed-loop system is lower than a specified upper bound.

Theorem 1 Consider 2D system (15) with $\boldsymbol{\omega}(t,k)=0$, the initial conditions Eq. (16) and the cost function Eq. (17), for some given positive scalars t and t_1, the robust guaranteed cost control problem of 2D system (9) is solvable if there exist positive definite matrices $\boldsymbol{S}>0$, $\boldsymbol{Y}>0$, $\bar{\boldsymbol{U}}_i>0$, $\boldsymbol{D}_{\mathrm{c}i}$, $\boldsymbol{Z}_i$, $\bar{\boldsymbol{Z}}_i$ and $\hat{\boldsymbol{Z}}_i$ ($i=1,2$) and positive scalars $\varepsilon_k\,(k=1,2)$ such that the following LMI holds

$$\begin{bmatrix}
-(t-t_1)\boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & t\boldsymbol{J}_{A_1}^{\mathrm{T}} & \boldsymbol{J}_{K_1}^{\mathrm{T}} & \boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_{\bar{F}}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{\beta}\boldsymbol{J}_{K_1}^{\mathrm{T}}\\
* & -t_1\boldsymbol{J}_{\mathrm{P}} & t\boldsymbol{J}_{A_2}^{\mathrm{T}} & \boldsymbol{J}_{K_2}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\beta}\boldsymbol{J}_{K_2}^{\mathrm{T}}\\
* & * & -t\boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_{\bar{E}} & \boldsymbol{0} & \boldsymbol{J}_{\bar{B}}\boldsymbol{\beta}_0 & \boldsymbol{0}\\
* & * & * & \boldsymbol{U}_3^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & -\bar{\boldsymbol{U}}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & -\bar{\boldsymbol{U}}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & -\varepsilon_1\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & -\varepsilon_1^{-1}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & -\varepsilon_2\boldsymbol{I} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & -\varepsilon_2^{-1}\boldsymbol{I}
\end{bmatrix}<0 \tag{18}$$

where $\boldsymbol{J}_{\mathrm{P}}=\begin{bmatrix}\boldsymbol{S} & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{Y}\end{bmatrix}$,

$$\boldsymbol{J}_{\bar{A}_i}=\begin{bmatrix}\boldsymbol{S}\boldsymbol{A}_i+\bar{\boldsymbol{Z}}_i\bar{\boldsymbol{C}} & \hat{\boldsymbol{Z}}_i\\ \boldsymbol{A}_i+\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{D}_{\mathrm{c}i}\bar{\boldsymbol{C}} & \boldsymbol{A}_i\boldsymbol{Y}+\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{Z}_i\end{bmatrix},$$

$$\boldsymbol{J}_{K_i}=\begin{bmatrix}\boldsymbol{D}_{\mathrm{c}i}\bar{\boldsymbol{C}} & \boldsymbol{Z}_i\end{bmatrix},$$

$$\boldsymbol{J}_{\bar{E}}=t\begin{bmatrix}\boldsymbol{S}\bar{\boldsymbol{E}}\\ \bar{\boldsymbol{E}}\end{bmatrix},$$

$$\boldsymbol{J}_{\bar{F}}=\begin{bmatrix}\bar{\boldsymbol{F}} & \bar{\boldsymbol{F}}\boldsymbol{Y}\end{bmatrix},$$

and $\boldsymbol{J}_{\bar{B}}=t\begin{bmatrix}\boldsymbol{S}\bar{\boldsymbol{B}}\\ \bar{\boldsymbol{B}}\end{bmatrix}$, the asterisk notation $(*)$ represents the symmetric element of a matrix.

Furthermore, system matrices $\boldsymbol{A}_{\mathrm{c}i}$, $\boldsymbol{B}_{\mathrm{c}i}$, $\boldsymbol{C}_{\mathrm{c}i}$, and $\boldsymbol{D}_{\mathrm{c}i}$ of the output feedback controller can be solved as

$$\begin{cases}\boldsymbol{A}_{\mathrm{c}i}=\bar{\boldsymbol{P}}_{12}^{-1}(\hat{\boldsymbol{Z}}_i-\boldsymbol{S}\boldsymbol{A}_i\boldsymbol{Y}-\bar{\boldsymbol{Z}}_i\bar{\boldsymbol{C}}\boldsymbol{Y}-\boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{C}_{\mathrm{c}i}\boldsymbol{P}_{12}^{\mathrm{T}})(\boldsymbol{P}_{12}^{\mathrm{T}})^{-1}\\ \boldsymbol{B}_{\mathrm{c}i}=\bar{\boldsymbol{P}}_{12}^{-1}(\bar{\boldsymbol{Z}}_i-\boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{D}_{\mathrm{c}i})\\ \boldsymbol{C}_{\mathrm{c}i}=(\boldsymbol{Z}_i-\boldsymbol{D}_{\mathrm{c}i}\bar{\boldsymbol{C}}\boldsymbol{Y})(\boldsymbol{P}_{12}^{\mathrm{T}})^{-1}\\ \boldsymbol{D}_{\mathrm{c}i}=\boldsymbol{D}_{\mathrm{c}i}\end{cases}\tag{19}$$

The cost function Eq. (17) of the resulting closed-loop 2D system $\sum_{\mathrm{2D-ep-F-o}}$ (15) satisfies

$$J\leqslant J^*=(t-t_1)r_2\lambda_{\max}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{M})+t_1r_1\lambda_{\max}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{M})\tag{20}$$

Proof Assume that there exist positive definite symmetric (PDS) matrixes $\boldsymbol{P}$ and $\boldsymbol{Q}$ such that

$$\begin{bmatrix}-(\boldsymbol{P}-\boldsymbol{Q}) & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{Q}\end{bmatrix}+\begin{bmatrix}\tilde{\boldsymbol{A}}_{1k}^{\mathrm{T}}\\ \tilde{\boldsymbol{A}}_2^{\mathrm{T}}\end{bmatrix}\boldsymbol{P}\begin{bmatrix}\tilde{\boldsymbol{A}}_{1k} & \tilde{\boldsymbol{A}}_2\end{bmatrix}+\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}<0\tag{21}$$

holds for all admissible actuator failures that satisfy Eq. (3), in which $\tilde{\boldsymbol{A}}_{1k}=\tilde{\boldsymbol{A}}_1+\tilde{\boldsymbol{\Delta}}_{\mathrm{a}}(t,k)$. Because $\boldsymbol{P}$ and $\boldsymbol{Q}$ are PDS matrixes, all functions $V_{\boldsymbol{P}}(\bullet)$, $V_{(\boldsymbol{P}-\boldsymbol{Q})>0}(\bullet)$ and $V_{\boldsymbol{Q}}(\bullet)$ satisfy conditions (1) and (2) of Lemma 1. Because $\boldsymbol{\omega}(t,k)=0$, we have

$$\begin{aligned}&V_{\boldsymbol{P}}[\boldsymbol{x}_{\mathrm{r}}(t+1,k)]-V_{\boldsymbol{P}-\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(t,k)]-V_{\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(t+1,k-1)]\\ &=V_{\left[\begin{smallmatrix}\tilde{\boldsymbol{A}}_{1k}^{\mathrm{T}}\\ \tilde{\boldsymbol{A}}_2^{\mathrm{T}}\end{smallmatrix}\right]\boldsymbol{P}\left[\begin{smallmatrix}\tilde{\boldsymbol{A}}_{1k} & \tilde{\boldsymbol{A}}_2\end{smallmatrix}\right]}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}-\\ &\quad V_{\left[\begin{smallmatrix}\boldsymbol{P}-\boldsymbol{Q} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{Q}\end{smallmatrix}\right]}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}\end{aligned}\tag{22}$$

From Eq. (21), we obtain

$$\begin{aligned}&V_{\left[\begin{smallmatrix}\tilde{\boldsymbol{A}}_{1k}^{\mathrm{T}}\\ \tilde{\boldsymbol{A}}_2^{\mathrm{T}}\end{smallmatrix}\right]\boldsymbol{P}\left[\begin{smallmatrix}\tilde{\boldsymbol{A}}_{1k} & \tilde{\boldsymbol{A}}_2\end{smallmatrix}\right]}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}-V_{\left[\begin{smallmatrix}\boldsymbol{P}-\boldsymbol{Q} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{Q}\end{smallmatrix}\right]}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}<\\ &-\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}\end{aligned}\tag{23}$$

Since $\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}>0$, the following inequality is effective

$$V_{\mathrm{P}}(\boldsymbol{x}_{\mathrm{r}}(t+1,k))<V_{\boldsymbol{P}-\boldsymbol{Q}}(\boldsymbol{x}_{\mathrm{r}}(t,k))+V_{\boldsymbol{Q}}(\boldsymbol{x}_{\mathrm{r}}(t+1,k-1))\tag{24}$$

Here $\Delta V=V_{\boldsymbol{P}}-V_{\boldsymbol{P}-\boldsymbol{Q}}-V_{\boldsymbol{Q}}$ denotes the function increment from a 2D view of energy transfer. As described in [17], the Lyapunov functional value clearly decreases along the state trajectories. From Definition 1, $\lim\limits_{t+k\to\infty}\bar{x}_{\mathrm{r}}(t,k)\to 0$ holds. Consequently, system (12) is asymptotically stable. Moreover, condition (3) in Lemma 1 is satisfied, which implies that the resulting closed-loop system (15) is fault-tolerant guaranteed cost control.

According to Lemma 3 in [26] and Schur complements [27], a sufficient condition for Eq. (21) is

$$\begin{bmatrix}-(\boldsymbol{P}-\boldsymbol{Q}) & \boldsymbol{0} & \tilde{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{K}_1^{\mathrm{T}} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \tilde{F}^T\\ * & -\boldsymbol{Q} & \tilde{\boldsymbol{A}}_2^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{K}_2^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & P\tilde{E} & \boldsymbol{0}\\ * & * & * & -\boldsymbol{U}_3^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & -\boldsymbol{U}_1^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & * & -\boldsymbol{U}_2^{-1} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & * & * & -\varepsilon_1\boldsymbol{I} & \boldsymbol{0}\\ * & * & * & * & * & * & * & -\varepsilon_1^{-1}\boldsymbol{I}\end{bmatrix}<0\tag{25}$$

Define $\boldsymbol{P}=t\boldsymbol{P}_1$, $\boldsymbol{Q}=t_1\boldsymbol{P}_1$, $\boldsymbol{\Omega}=\boldsymbol{P}_1^{-1}$, and pre- and post-multiply the left-hand side matrix in Eq. (25) by the matrix $\mathrm{diag}[\boldsymbol{\Omega},\boldsymbol{\Omega},\boldsymbol{\Omega},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}]$ separately. Partition $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}^{-1}$ as follows

$$\boldsymbol{\Omega}=\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{P}_{12}\\ \boldsymbol{P}_{12}^{\mathrm{T}} & \boldsymbol{P}_{22}\end{bmatrix},\quad \boldsymbol{\Omega}^{-1}=\begin{bmatrix}\boldsymbol{S} & \bar{\boldsymbol{P}}_{12}\\ \bar{\boldsymbol{P}}_{12}^{\mathrm{T}} & \bar{\boldsymbol{P}}_{22}\end{bmatrix}$$

where $\boldsymbol{S}, \boldsymbol{Y}, \boldsymbol{P}_{12}, \bar{\boldsymbol{P}}_{12} \in \boldsymbol{R}^{n\times n}$, and $\boldsymbol{P}_{12}\bar{\boldsymbol{P}}_{12}^{T}=\boldsymbol{I}-\boldsymbol{YX}$.

Let $\boldsymbol{J}=\begin{bmatrix}\boldsymbol{S} & \boldsymbol{I}\\ \bar{\boldsymbol{P}}_{12}^{\mathrm{T}} & \boldsymbol{0}\end{bmatrix}$, $\bar{\boldsymbol{J}}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{Y}\\ \boldsymbol{0} & \boldsymbol{P}_{12}^{\mathrm{T}}\end{bmatrix}$, and we have

$$\boldsymbol{\Omega J}=\bar{\boldsymbol{J}},\ \boldsymbol{J}^{\mathrm{T}}\boldsymbol{\Omega J}=\begin{bmatrix}\boldsymbol{S} & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{Y}\end{bmatrix},$$

$$\boldsymbol{J}^{\mathrm{T}}\tilde{\boldsymbol{A}}_i\boldsymbol{\Omega J}=\begin{bmatrix}\boldsymbol{SA}_i+\bar{\boldsymbol{Z}}_i\bar{\boldsymbol{C}} & \hat{\boldsymbol{Z}}_i\\ \boldsymbol{A}_i+\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{D}_{ci}\bar{\boldsymbol{C}} & \boldsymbol{A}_i\boldsymbol{Y}+\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{Z}_i\end{bmatrix}+\begin{bmatrix}\boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\alpha}_0\boldsymbol{\beta}\boldsymbol{D}_{ci}\bar{\boldsymbol{C}} & \boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\alpha}_0\boldsymbol{\beta}\boldsymbol{Z}_i\\ \bar{\boldsymbol{B}}\boldsymbol{\alpha}_0\boldsymbol{\beta}\boldsymbol{D}_{ci}\bar{\boldsymbol{C}} & \bar{\boldsymbol{B}}\boldsymbol{\alpha}_0\boldsymbol{\beta}\boldsymbol{Z}_i\end{bmatrix},$$

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\Omega}\tilde{\boldsymbol{F}}^{\mathrm{T}}=\begin{bmatrix}\bar{\boldsymbol{F}}^{\mathrm{T}}\\ \boldsymbol{Y}\bar{\boldsymbol{F}}^{\mathrm{T}}\end{bmatrix},\ \boldsymbol{J}^{\mathrm{T}}\tilde{\boldsymbol{E}}=\begin{bmatrix}\boldsymbol{S}\bar{\boldsymbol{E}}\\ \bar{\boldsymbol{E}}\end{bmatrix},$$ in which

$$\hat{\boldsymbol{Z}}_i=\boldsymbol{SA}_i\boldsymbol{Y}+\bar{\boldsymbol{Z}}_i\bar{\boldsymbol{C}}\boldsymbol{Y}+\boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{C}_{ci}\boldsymbol{P}_{12}^{\mathrm{T}}+\bar{\boldsymbol{P}}_{12}\boldsymbol{A}_{ci}\boldsymbol{P}_{12}^{\mathrm{T}},$$

$$\boldsymbol{Z}_i=\boldsymbol{D}_{ci}\bar{\boldsymbol{C}}\boldsymbol{Y}+\boldsymbol{C}_{ci}\boldsymbol{P}_{12}^{\mathrm{T}}\ \text{and}\ \bar{\boldsymbol{Z}}_i=\boldsymbol{S}\bar{\boldsymbol{B}}\boldsymbol{\beta}\boldsymbol{D}_{ci}+\bar{\boldsymbol{P}}_{12}\boldsymbol{B}_{ci}.$$

Meanwhile, pre- and post-multiply the nonsingular matrix $\boldsymbol{\varUpsilon}^{\mathrm{T}}$ and diag $\boldsymbol{\varUpsilon}=\mathrm{diag}[\boldsymbol{J}\ \boldsymbol{J}\ \boldsymbol{J}\ \boldsymbol{I}\ \boldsymbol{J}\ \boldsymbol{J}\ \boldsymbol{I}\ \boldsymbol{I}]$ in Eq. (25), and let $\boldsymbol{J}^{\mathrm{T}}\boldsymbol{U}_i^{-1}\boldsymbol{J}=\bar{\boldsymbol{U}}_i$ ($i=2,3$). By using Lemma 3 in [26] and Schur complements [27], Eq. (25) is equivalent to Eq. (18).

Since inequality (21) holds, we have

$$\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\Delta V[\boldsymbol{x}_{\mathrm{r}}(t,k)]\leqslant\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}-\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}\tag{26}$$

It follows from Eq. (26) and the definitions $\boldsymbol{Q}$ and $\boldsymbol{P}$ that

$$\begin{aligned}&(t-t_1)\sum_{k=1}^{N_2}[V_{P_1}(N_1,k)-V_{P_1}(0,k)]+t_1\sum_{t=0}^{N_1}[V_{P_1}(t+1,N_2)-V_{P_1}(t+1,0)]\\ &\leqslant\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}-\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}U_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}\end{aligned}\tag{27}$$

For N_1, $N_2\to\infty$, it follows from Definition 1, the definitions $\boldsymbol{P}$ and $\boldsymbol{Q}$, and Eq. (16) that

$$\begin{aligned}&\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2\end{bmatrix}\begin{pmatrix}\boldsymbol{x}_{\mathrm{r}}(t,k)\\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1)\end{pmatrix}\leqslant(t-t_1)\sum_{k=1}^{N_2}V_{P_1}(0,k)+t_1\sum_{t=0}^{N_1}V_{P_1}(t+1,0)\\ &\leqslant(t-t_1)\sum_{k=1}^{N_2}\boldsymbol{x}_{\mathrm{r}}^{\mathrm{T}}(0,k)\boldsymbol{\Omega}^{-1}\boldsymbol{x}_{\mathrm{r}}(0,k)+t_1\sum_{t=0}^{N_1}\boldsymbol{x}_{\mathrm{r}}^{\mathrm{T}}(t+1,0)\boldsymbol{\Omega}^{-1}\boldsymbol{x}_{\mathrm{r}}(t+1,0)\leqslant(t-t_1)r_2\lambda_{\max}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{SM})+t_1r_1\lambda_{\max}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{SM})\end{aligned}\tag{28}$$

This completes the proof.

Theorem 2 Consider 2D system (15), initial conditions (16) and cost function (17). For some given positive scalars t and t_1, the robust H_∞ fault-tolerant guaranteed cost control problem of 2D system (15) is solvable *via* a 2D output feedback controller Eq. (14) with system matrices satisfying Eq. (19) if there exist matrices $\boldsymbol{S}>0$, $\boldsymbol{Y}>0$, $\bar{\boldsymbol{U}}_i>0$, $\boldsymbol{D}_{ci}$, $\boldsymbol{Z}_i$, $\bar{\boldsymbol{Z}}_i$ and $\hat{\boldsymbol{Z}}_i$ ($i=1,2$) and positive scalars $\varepsilon_k(k=1,2)$ and γ such that the following LMI holds

$$\begin{bmatrix}
-(t-t_1)\boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & t\boldsymbol{J}_{A_1}^{\mathrm{T}} & \boldsymbol{J}_{K_1}^{\mathrm{T}} & \boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_{\bar{F}}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{\beta}\boldsymbol{J}_{K_1}^{\mathrm{T}} & \boldsymbol{J}_H\\
* & -t_1\boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & t\boldsymbol{J}_{A_2}^{\mathrm{T}} & \boldsymbol{J}_{K_2}^{\mathrm{T}} & \boldsymbol{0} & J_P & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\beta}\boldsymbol{J}_{K_2}^{\mathrm{T}} & \boldsymbol{0}\\
* & * & -\gamma\boldsymbol{I} & t\boldsymbol{J}_G^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & -t\boldsymbol{J}_{\mathrm{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_{\bar{E}} & \boldsymbol{0} & \boldsymbol{J}_{\bar{B}}\boldsymbol{\beta}_0 & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & -\boldsymbol{U}_3^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & -\bar{\boldsymbol{U}}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & -\bar{\boldsymbol{U}}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & -\varepsilon_1\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & -\varepsilon_1^{-1}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & -\varepsilon_2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & * & -\varepsilon_2^{-1}\boldsymbol{I} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & * & * & -\gamma\boldsymbol{I}
\end{bmatrix}<0\tag{29}$$

where $\boldsymbol{J}_{\boldsymbol{G}}^{\mathrm{T}}=\left[\boldsymbol{G}^{\mathrm{T}}\boldsymbol{S} \quad \boldsymbol{G}^{\mathrm{T}}\right]$, $\boldsymbol{J}_{\boldsymbol{H}}^{\mathrm{T}}=\left[\boldsymbol{H} \quad \boldsymbol{HY}\right]$, and others are designed in Theorem 1. In this case, the robust H_∞ reliable guaranteed cost control law can be still chosen as Eq. (19) and the corresponding cost function of the resulting closed-loop 2D system $\sum_{\text{2D-ep-F-c}}$ (15) still satisfies Eq. (20).

Proof Assume that there exist PDS matrixes $\boldsymbol{P}$ and $\boldsymbol{Q}$, a scalar $\gamma>0$ such that

$$\begin{bmatrix} -(\boldsymbol{P}-\boldsymbol{Q})+\gamma^{-1}\tilde{\boldsymbol{H}}^{\mathrm{T}}\tilde{\boldsymbol{H}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\gamma\boldsymbol{I} \end{bmatrix}+\begin{bmatrix} \tilde{\boldsymbol{A}}_{1k}^{\mathrm{T}} \\ \tilde{\boldsymbol{A}}_{2}^{\mathrm{T}} \\ \tilde{\boldsymbol{G}}^{\mathrm{T}} \end{bmatrix}\boldsymbol{P}\left[\tilde{\boldsymbol{A}}_{1k} \quad \tilde{\boldsymbol{A}}_{2} \quad \tilde{\boldsymbol{G}}\right]+\begin{bmatrix} \boldsymbol{U}_1+\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2 & \mathbf{0} \\ \boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_1 & \boldsymbol{U}_2+\boldsymbol{K}_2^{\mathrm{T}}\boldsymbol{U}_3\boldsymbol{K}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}<0 \tag{30}$$

holds for all admissible actuator failures satisfying Eq. (3). For any nonzero $\boldsymbol{\omega}(t,k)\in l_2\{[0,\infty],[0,\infty]\}$, we define

$$J_1=\Delta V[\boldsymbol{x}_{\mathrm{r}}(t,k)]+\gamma^{-1}\boldsymbol{z}^{\mathrm{T}}(t,k)\boldsymbol{z}(t,k)-\gamma\boldsymbol{\omega}^{\mathrm{T}}(t,k)\boldsymbol{\omega}(t,k) \tag{31}$$

where

$$\Delta V[\boldsymbol{x}_{\mathrm{r}}(t,k)]=V_{\boldsymbol{p}}[\boldsymbol{x}_{\mathrm{r}}(t+1,k)]-V_{\boldsymbol{P}-\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(t,k)]-V_{\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(t+1,k-1)] \tag{32}$$

According to Eq. (15), we have

$$J_1=V_{\begin{bmatrix} \tilde{\boldsymbol{A}}_{1k}^{\mathrm{T}} \\ \tilde{\boldsymbol{A}}_{2}^{\mathrm{T}} \\ \tilde{\boldsymbol{G}}^{\mathrm{T}} \end{bmatrix}\boldsymbol{P}\left[\tilde{\boldsymbol{A}}_{1k} \quad \tilde{\boldsymbol{A}}_{2} \quad \tilde{\boldsymbol{G}}\right]-\begin{bmatrix} (\boldsymbol{P}-\boldsymbol{Q})-\gamma^{-1}\tilde{\boldsymbol{H}}^{\mathrm{T}}\tilde{\boldsymbol{H}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \gamma\boldsymbol{I} \end{bmatrix}}\begin{bmatrix} \boldsymbol{x}_{\mathrm{r}}(t,k) \\ \boldsymbol{x}_{\mathrm{r}}(t+1,k-1) \\ \boldsymbol{w}(t,k) \end{bmatrix} \tag{33}$$

For any integers M_1, $M_2>0$ according to the assumption that all boundary conditions of system $\sum_{\text{2D-ep-F-o}}$ are zero, with Eq. (32), we have

$$\sum_{t=0}^{M_1}\sum_{k=1}^{M_2}\Delta V[\boldsymbol{x}_{\mathrm{r}}(t,k)]=\sum_{t=0}^{M_1}\sum_{k=1}^{M_2}\left[V_{\boldsymbol{p}}[\boldsymbol{x}_{\mathrm{r}}(t+1,k)]-V_{\boldsymbol{P}-\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(t,k)]-V_{\boldsymbol{Q}}[x_{\mathrm{r}}(t+1,k-1)]\right]$$
$$=\sum_{k=1}^{M_2-1}V_{\boldsymbol{p}-\boldsymbol{Q}}[\boldsymbol{x}_{\mathrm{r}}(M_1+1,k)]+V_{\boldsymbol{p}}[\boldsymbol{x}_{\mathrm{r}}(M_1+1,M_2)]\geqslant 0 \tag{34}$$

Hence, for any nonzero $\boldsymbol{\omega}(t,k)\in l_2\{[0,\infty],[0,\infty]\}$,

$$\sum_{t=0}^{M_1}\sum_{k=1}^{M_2}\left[\gamma^{-1}\boldsymbol{z}^{\mathrm{T}}(t,k)\boldsymbol{z}(t,k)-\gamma\boldsymbol{\omega}^{\mathrm{T}}(t,k)\boldsymbol{\omega}(t,k)\right]$$
$$\leqslant\sum_{t=0}^{M_1}\sum_{k=1}^{M_2}J_1=\sum_{t=0}^{M_1}\sum_{k=1}^{M_2}\left[\gamma^{-1}\boldsymbol{z}^{\mathrm{T}}(t,k)\boldsymbol{z}(t,k)-\gamma\boldsymbol{\omega}^{\mathrm{T}}(t,k)\boldsymbol{\omega}(t,k)+\Delta V[\boldsymbol{x}_{\mathrm{r}}(t,k)]\right]<0 \tag{35}$$

i.e., $\|\boldsymbol{z}(t,k)\|_{\text{2D-2e}}\leqslant\gamma\|\boldsymbol{\omega}(t,k)\|_{\text{2D-2e}}$. Moreover, along the lines similar to the proof of Theorem 1, it is easy to obtain Eq. (29).

3.3 Procedure design

For any $N_1\geqslant r_1$ and $N_2\geqslant r_2$, the boundary conditions satisfy Eq. (16) and from Theorem 1, there exists $\beta>0$ such that the cost bound Eq. (20) leads to

$$J\leqslant r_2\left(t-t_1\right)\beta+r_1t_1\beta \tag{36}$$

where

$$\begin{bmatrix} -\beta\boldsymbol{I} & \boldsymbol{\Theta}^{\mathrm{T}} \\ \boldsymbol{\Theta} & -\boldsymbol{S} \end{bmatrix}<0 \tag{37}$$

In order to obtain the output controller and achieve as far as possible the least guaranteed cost value J^*, we have to solve the following optimization problem

$$\min \quad r_2\left(t-t_1\right)\beta+r_1t_1\beta$$
$$\text{s.t.} \quad t-t_1>0,\ t_1>0,\ (18),\ (37) \tag{38}$$

With a similar line as in [28] to propose a nonlinear minimization problem involving LMI conditions and utilize the linearization method [29], when positive scalars t and t_1 are given, the ideal values of which can be obtained by using the following method: given larger t and t_1, solve inequality (18); if there is a feasible solution, given smaller t and t_1, go on; otherwise stop; the above optimization problem is a convex optimization problem, which can be solved by the solver Mincx in the LMI toolbox.

Similar to Theorem 1, the optimization problem of Theorem 2 is expressed as

$$\min \quad r_2\left(t-t_1\right)\beta+r_1t_1\beta$$
$$\text{s.t.} \quad t-t_1>0,\ t_1>0,\ (29),\ (37) \tag{39}$$

In this paper, in order to obtain the minimum guaranteed cost bound, there is no constraint to γ. The value of γ can be obtained by solving Eq. (29).

4 ILLUSTRATION

In this section, injection molding, a typical batch process, will be used as example for illustration. Injection molding mainly consists of 3 phases: filling, packing, and cooling [9]. For the packing/holding phase, nozzle pressure is a key process variable that should be controlled to follow a preset profile to ensure product quality and consistency from cycle to cycle. Variations of working conditions may make injection molding particularly packing-holding viewed as a batch process with uncertainties. In each cycle, the transition of different phases leads to uncertain initial values of the nozzle pressure. This makes the conventional ILC inapplicable. Moreover, the control performance is poor when a slow hydraulic valve is used. Pure feedback control cannot improve control performance from cycle to cycle. It is necessary to design a controller that can improve both the performance over time and the tracking performance from cycle to cycle. Based on the open-loop test and analysis, to identify the nozzle packing pressure response to the hydraulic control valve opening, the state-space mode is considered as the state variables [20]

$$\Sigma_{\mathrm{P}}:\ \boldsymbol{x}(t+1,k)=\left(\begin{bmatrix}1.607 & -0.6086\\ 1 & 0\end{bmatrix}+\boldsymbol{\Delta A}\right)\boldsymbol{x}(t,k)+\begin{bmatrix}1.239\\ -0.9282\end{bmatrix}\boldsymbol{u}(t,k)$$
$$\boldsymbol{y}(t,k)=\begin{bmatrix}1 & 0\end{bmatrix}\boldsymbol{x}(t,k) \tag{40}$$

where batch-to-batch time-varying uncertainties of the state transfer matrices are expressed as $\boldsymbol{\Delta A}=\begin{bmatrix}0.03\delta_1 & 0.04\delta_2\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0.03 & 0.04\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_1 & 0\\ 0 & \delta_2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $|\delta_i|\leqslant 1,\ i=1,2$. Assuming that there exists an unknown actuator failure α, we know that $0.8=\underline{\alpha}\leqslant\alpha\leqslant\overline{\alpha}=1$. Using Eq. (6), $\beta=0.9$ and $\beta_0=0.1$ are obtained. Here the set-point takes the form of

$$y_{\mathrm{d}}(t)=15,\quad (\text{for } 0\leqslant t\leqslant 100) \tag{41a}$$

$$y_{\mathrm{d}}(t)=30,\quad (\text{for } 100\leqslant t\leqslant 200) \tag{41b}$$

The initial state satisfies condition (16) for $r_1=r_2=10$, and belongs to the set S where

$$\boldsymbol{\Theta}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \tag{42}$$

We choose the weighting matrices

$$\boldsymbol{U}_1=\boldsymbol{U}_2=\boldsymbol{\Theta},\quad U_3=1. \tag{43}$$

Solving the problem described by Eqs. (38) and (39), when $t=0.75$ and $t_1=0.2$, the corresponding controllers are achieved for repetitive perturbations and nonrepetitive perturbations, as shown in Tables 1 and 2.

The least upper bounds of the corresponding closed-loop cost function are $J^*=1.0232\times10^4$ and $J^*=1.244\times10^4$. To show tracking results, we only choose the ILRGCC in Table 1, *i.e*., repetitive perturbations, to stabilize system (40), and use the output tracking error in terms of root-sum-squared-error (REES) criterion. The results are shown in Figs. 2 and 3, here $\{\delta_i:|\delta_i|<1\}_{i=1,2,3}$ are assumed to vary with time randomly within $[0,1]$. The tracking performance is improved from cycle to cycle, although after the fault occurs, the tracking performance experiences degradation, which can be also seen by the least cost function J^*. The tracking performance can achieve a perfect level again some cycles later, even return to the original level.

Table 1 Design results for repetitive perturbations ($t=0.75$, $t_1=0.2$)

	$\boldsymbol{A}_{ci}$	$\boldsymbol{B}_{ci}$	$\boldsymbol{C}_{ci}$	$\boldsymbol{D}_{ci}$
$i=1$	$\begin{bmatrix}-0.3036 & 0.0000 & 00.0000\\ -3.4010 & 0.8005 & 0.0000\\ 10.6030 & -3.8231 & -0.1590\end{bmatrix}$	$\begin{bmatrix}0.1500 & 0.0000\\ 1.1552 & -1.2616\\ -8.1105 & 5.4750\end{bmatrix}$	$\begin{bmatrix}-2.5300 & 0.7030 & -0.0000\end{bmatrix}$	$\begin{bmatrix}1.5604 & -1.4174\end{bmatrix}$
$i=2$	$\begin{bmatrix}0.1097 & 0.0000 & 0.0000\\ 0.2801 & 0.0000 & 0.0000\\ 10.0006 & -0.0300 & -0.0225\end{bmatrix}$	$\begin{bmatrix}-0.0400 & 0.6488\\ -0.1036 & 1.0772\\ -3.0751 & -2.0033\end{bmatrix}$	$\begin{bmatrix}0.0000 & 0.0000 & -0.0010\end{bmatrix}$	$\begin{bmatrix}0.0000 & -0.5370\end{bmatrix}$

Table 2 Design results for nonrepetitive perturbations ($t=0.75$, $t_1=0.2$, $\gamma=20.5$)

	$\boldsymbol{A}_{ci}$	$\boldsymbol{B}_{ci}$	$\boldsymbol{C}_{ci}$	$\boldsymbol{D}_{ci}$
$i=1$	$\begin{bmatrix}-0.2001 & -0.0000 & -0.0000\\ -3.4003 & 0.9000 & 0.0000\\ 6.7857 & 0.0000 & -0.0137\end{bmatrix}$	$\begin{bmatrix}0.0000 & 0.1481\\ 1.5032 & -1.1952\\ -18.1167 & 5.4032\end{bmatrix}$	$\begin{bmatrix}-2.2530 & 0.5911 & 0.0000\end{bmatrix}$	$\begin{bmatrix}1.7544 & -1.4165\end{bmatrix}$
$i=2$	$\begin{bmatrix}0.1244 & -0.0000 & 0.0000\\ 0.2933 & -0.0000 & 0.0000\\ 7.4879 & -0.0001 & -0.0600\end{bmatrix}$	$\begin{bmatrix}-0.0000 & 0.7012\\ -0.0000 & 1.2072\\ -1.0751 & -16.1344\end{bmatrix}$	$\begin{bmatrix}0.0000 & -0.0000 & -0.0000\end{bmatrix}$	$\begin{bmatrix}-0.0000 & -0.5533\end{bmatrix}$

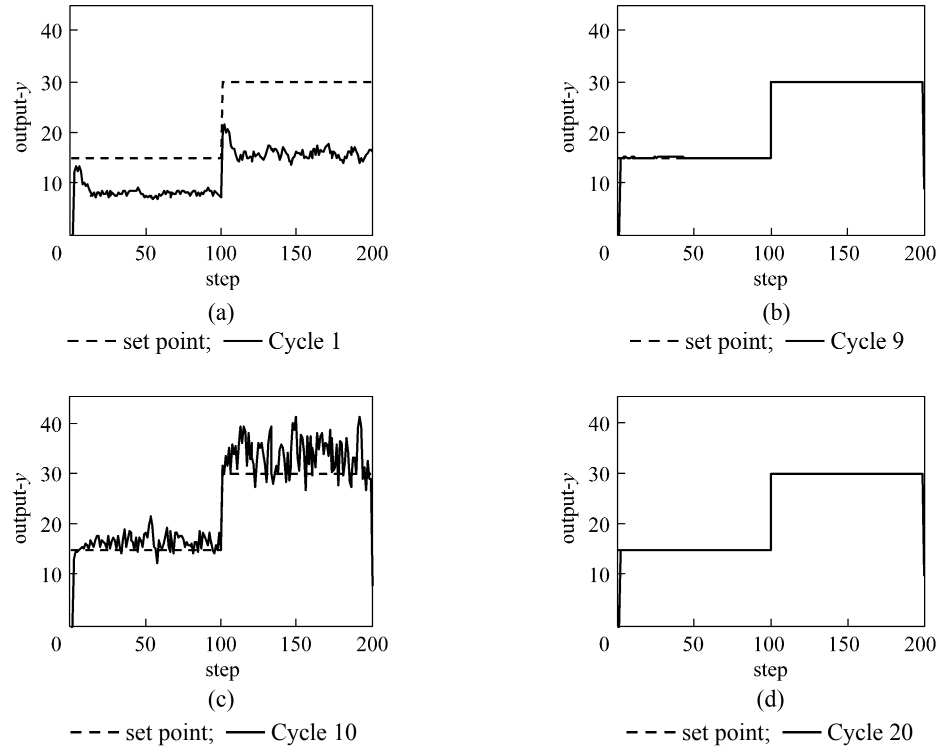

Figure 2 Output responses in repetitive case

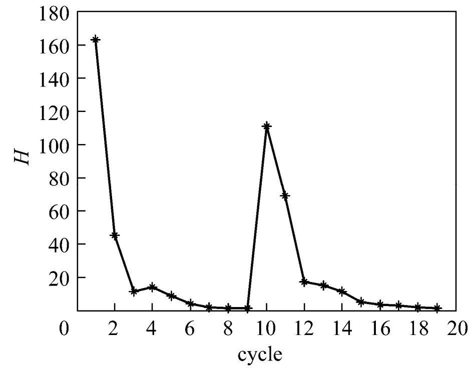

Figure 3 Tracking performances in repetitive case

5 CONCLUSIONS

By an LMI framework, the optimal fault-tolerant guaranteed cost control problem *via* 2D-ILRGCC is proposed for a batch process with actuator failures. The process is transformed to an equivalent 2D-FM model, based on which relevant concepts on the ILRGCC design is presented. Through solving the corresponding LMI constraints, the controller is explicitly formulated, with preserving the least guaranteed cost and H_∞ performance index. The proposed 2D-LRGCC can guarantee control performance improvement not only along the time direction but also along the cycle direction, even with actuator failures. An injection pressure control is developed to demonstrate the effectiveness and merits of the proposed ILC method.

REFERENCES

1 Korovessi, E., Linninger, A.A., *Batch Processes*, CRC/Taylor and Francis, Boca Raton, FL (2006).

2 Anon., "Automatic control of batch-process temperature", *Food Ind*., **8** (5), 237–244 (1936).

3 Yu, J., Qin, S.J., "Multiway Gaussian mixture model based umlti-phase batch process monitoring", *Ind. Eng. Chem. Res.*, **48**, 8585–8594 (2009).

4 Zhao, C.H., Mo, S.Y., Gao, F.R., Lu, N.Y., Yao, Y., "Statistical analysis and online monitoring for handling multiphase batch processes with varying durations", *Journal of Process Control*, **21**, 817–829 (2011).

5 Ye, D., Yang, G.H., "Delay-dependent reliable H_∞ control for linear time-varying delay systems *via* adaptive approach", *American Control Conference*, 3991–3996 (2007).

6 Wang, Y.Q., Zhou, D., Gao, F., "Iterative learning fault-tolerant control for batch processes", *Ind. Eng. Chem. Res.*, **45**, 9050–9060 (2006).

7 Wang, Y.Q., Zhou, D., Gao, F., "Iterative learning reliable control of batch processes with sensor faults", *Control Engineering Science*, **63**, 1039–1051 (2008).

8 Wang, Y.Q., Zhou, D., Gao, F., "Active fault-tolerant control of nonlinear batch processes with sensor faults", *Ind. Eng. Chem. Res.*, **46**, 9158–9169 (2007).

9 Gao, F., Yang, Y., Shao, C., "Robust iterative learning control with applications to injection molding process", *Chem. Eng. Sci.*, **56**, 7025–7034 (2001).

10 Xiong, Z., Zhang, J., "Product quality trajectory tracking in batch processes using iterative learning control based time-varying perturbation models", *Ind. Eng. Chem. Res.*, **42** (26), 6802–6814 (2003).

11 Zhang, R.D., Wang, S.Q., "Support vector machine based predictive functional control design for output temperature of coking furnace",

J. Process Control, **18**, 439−448 (2008).

12 Zhang, R.D., Li, P., Xue, A. K., Jiang, A. P., Wang, S.Q., "A simplified linear iterative predictive functional control approach for chamber pressure of industrial coke furnace", *J. Process Control*, **20**, 464−471 (2010).

13 Zhang, R.D., Xue, A. K., Wang, S.Q., "Dynamic modeling and nonlinear predictive control based on partitioned model and nonlinear optimization", *Ind. Eng. Chem. Res.*, **50** (13), 8110−8121 (2011).

14 Liu, T., Gao, F., "Robust two-dimensional iterative learning control for batch processes with state delay and time-varying uncertainties", *Chemical Engineering Science*, **65**, 6134−6144 (2010).

15 Shi, J., Gao, F., Wu, T.J., "Two-dimensional linear quadratic optimal controls and system analysis", *Ind. Eng. Chem. Res.*, **45**, 4603−4616 (2006).

16 Shi, J., Gao, F., Wu, T.J., "Iterative learning controls for batch processes", *Ind. Eng. Chem. Res.*, **45**, 4617−4628 (2006).

17 Shi, J., Gao, F., Wu, T.J., "Single-cycle and multi-cycle generalized 2D model predictive iterative learning control (2D-GPILC) schemes for batch processes", *J. Process Control*, **17**, 715−727 (2007).

18 Shi, J., Gao, F., Wu, T.J., "Robust design of integrated feedback and iterative learning control of a batch process based on a 2D Roesser system", *J. Process Control*, **15**, 907−924 (2005).

19 Shi, J., Gao, F., Wu, T.J., "A robust iterative learning control design for batch processes with uncertain perturbation and initialization", *AIChE* J., **52** (6), 2171−2187 (2007).

20 Shi, J., Gao, F., Wu, T.J., "Integrated design and structure analysis of robust iterative learning control system based on a two-dimensional model", *Ind*. *Eng*. *Chem*. *Res*., **44**, 8095−8105 (2005).

21 Chang, S.S.L., Peng, T.K.C., "Adaptive guaranteed cost control of systems with uncertain parameters", *IEEE Trans. Automat. Control*, **17** (4), 474−483 (1972).

22 Dhawan, A., Kar, H., "Optimal guaranteed cost control of 2-D discrete uncertain systems: An LMI approach", *Signal Processing*, **87**, 3075−3085 (2007).

23 Dhawan, A., Kar, H., "An LMI approach to robust optimal guaranteed cost control of 2-D discrete systems described by the Roesser model", *Signal Processing*, **90**, 2648−2654 (2010).

24 Dhawan, A., Kar, H., "An improved LMI-based criterion for the design of optimal guaranteed cost controller for 2-D discrete uncertain systems", *Signal Processing*, **91**, 1032−1035 (2011).

25 Kaczorek, T., Two-Dimensional Linear Systems, Springer-Verlag, Berlin, Germany (1985).

26 Yu, K.W., Lien, C.H., "Stability criteria for uncertain neutral systems with interval time-varying delays", *Chaos, Solitons and Fractals*, **38**, 650−657 (2008).

27 Boyd, S., Ghaoui, EI, L., Feron, E., Balakrishnan, V., Linear Matrix Inequality in System and Control Theory, Society for Industrial and Applied Mathematics, Philadelphia, USA (1994).

28 Ghaoui, L.E., Oustry, F., AitRami, M., "A cone complementarity linearization algorithm for static output-feedback and related problems", *IEEE Transactions on Automatic Control*, **42**, 1171−1176 (1997).

29 Moon Y.S., Park, P., Kwon, W.H., Lee, Y.S., "Delay-dependent robust stabilization of uncertain state-delayed systems", *International Journal of control*, **74**, 1447−1455 (2001).