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Bifurcation control and eigenstructure assignment in continuous solution polymerization of vinyl acetate[☆]

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ABSTRACT

The major difficulty in achieving good performance of industrial polymerization reactors lies in the lack of understanding of their nonlinear dynamics and the lack of well-developed techniques for the control of nonlinear processes, which are usually accompanied with bifurcation phenomenon. This work aims at investigating the nonlinear behavior of the parameterized nonlinear system of vinyl acetate polymerization and further modifying the bifurcation characteristics of this process via a washout filter-aid controller, with all the original steady state equilibria preserved. Advantages and possible extensions of the proposed methodology are discussed to provide scientific guide for further controller design and operation improvement.

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1. Introduction

Most of polymerization reactions are highly coupled with exothermic processes [1] and diffusion limitations such as gel effect [2], so very interesting and troublesome nonlinear dynamics may arise in most polymerization processes. For monomers such as ethylene, methylmethacrylate, vinyl acetate and styrene, polymerization reactors have presented some interesting dynamic behavior and oscillations theoretically and experimentally [1–8].

Bifurcations also occur under traditional state feedback and/or adaptive controls. Even worse, sometimes the local instability and complex dynamical behavior can result from controlled systems [9]. In these situations, adequate process dynamics and stability boundary are not available, so one or more poles of closed-loop transfer function of linearized system may move across the stability boundary and lead to potential divergence and oscillation [10]. Moreover, state feedback and/or adaptive controls will change the location of particular operating point on the steady state equilibrium and shapes of other equilibria. This will not only waste control energy associated with the forced alteration of the system equilibrium structure [11], but also pose difficulties in controller design that emphasizes the performance improvement by a particular preferable operating point. To overcome these problems,

Abed and Fu [12,13] were among the first to analyze the possibility of bifurcation control and further proposed the technique of washout filter-aided dynamic controllers [10] in the cases with uncertain targeted operating point. Although the washout filters have been successfully used in many control applications, there is no systematic ways of choosing their constants and control parameters since this technique is based on eigenstructure assignment. A series of papers [2–8] contribute to the model development and dynamic behavior of polymerization reactors, but from a practical point of view, the dynamic structures presented therein have only theoretical importance for the reactor since most of them lead to unstable oscillations and are confined to narrow ranges of operation conditions [5]. No strategy has been proposed to deal with local unstable branch where particular targeted operating points may locate.

It is the purpose of this paper to investigate the nonlinear behavior of the parameterized nonlinear system of continuous solution polymerization of vinyl acetate (VA) in *t*-butanol with 2,2'-azoisobutyronitrile (AIBN) as initiator. A technique of bifurcation control is adopted to modify the bifurcation characteristics of this process via a washout filter-aid controller with the preservation of all desired equilibrium points of the open-loop system.

2. Reactor Model of Vinyl Acetate Polymerization

In this study on a continuous process of solution polymerization of vinyl acetate (VA) in *t*-butanol with 2,2'-azoisobutyronitrile (AIBN) as initiator, the free-radical kinetic mechanism is adopted, including

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chemical initiation, propagation and termination and chain transfer to the monomer. The mathematical model herein is a modification of that proposed by Teymour and Ray [6] for a full scale continuous stirred tank reactor (CSTR). Based on the reaction mechanism shown in Table 1, four ordinary differential equations for continuous solution polymerization of vinyl acetate are given in Eqs. (1) to (4) in Table 2, with regard to the monomer volume fraction v_m , solvent volume fraction v_s , dimensionless initiator concentration z_i , and dimensionless reactor temperature y_T .

Table 1
Kinetic mechanism for continuous solution polymerization of vinyl acetate

Description	Step
Initiation	$I \xrightarrow{k_d} 2R \cdot$
Initiation	$R \cdot + M \xrightarrow{k_p} P_1 \cdot$
Propagation	$P_n \cdot + M \xrightarrow{k_p} P_{n+1} \cdot$
Chain transfer	$P_n \cdot + M \xrightarrow{k_{trm}} P_1 \cdot + M_n$
Termination	$P_n \cdot + P_m \cdot \xrightarrow{k_t} \text{dead polymer}$

Table 2
Mathematical model for solution polymerization of vinyl acetate in a CSTR

$\frac{dv_m}{d\tau} = (1 - \varphi_s) \frac{\rho_{mf}}{\rho_m} - v_m \left(\frac{q_{out}}{q_{in}} \right) - \theta v_m (k_p + k_{trm}) P - \theta \frac{[MW]_m}{\rho_m} 2fk_d z_i I_f - \frac{v_m}{\rho_m} \frac{d\rho_m}{d\tau}$	(1)
$\frac{dv_s}{d\tau} = \varphi_s \frac{\rho_{sf}}{\rho_s} - v_s \left(\frac{q_{out}}{q_{in}} \right) - \frac{v_s}{\rho_s} \frac{d\rho_s}{d\tau}$	(2)
$\frac{dz_i}{d\tau} = 1 - \theta k_d z_i - z_i \left(\frac{q_{out}}{q_{in}} \right)$	(3)
$\frac{dy_T}{d\tau} = \left[\frac{\rho_f C_{pf}}{\rho_p C_{pc}} (y_f - y_{ref}) - \frac{\rho_f}{\rho_p} (y_f - y_{ref}) + \frac{-\Delta H_p}{\rho_p C_{pc}} \theta v_m k_p \frac{\rho_p}{[MW]_m} P - \theta \frac{\alpha B}{\rho_p} (y - 1) \right]$	(4)
$q_{out} = q_{in} \left\{ \frac{\rho_f}{\rho_{f(T)}} + \theta [MW]_m R_m \left[\frac{1}{\rho_p} - \frac{1}{\rho_m} \right] - \sum_i \frac{v_i}{\rho_i} \frac{d\rho_i}{d\tau} \right\}$	(5)
$P = \sqrt{\frac{2fk_d z_i I_f}{k_t}}$	(6)
where	
$\tau = \frac{t}{\theta}$	(7)
$z_i = \frac{I_f}{I_f}$	(8)
$y = \frac{T}{T_c}$	(9)
$y_f = \frac{T_f}{T_c}$	(10)

Eq. (5) describes the reactor outlet flow rate q_{out} , while Eq. (6) gives the expression of live radical concentration P using the quasi-steady-state assumption. Note that the method of moments could be adopted to derive the moment equations for living radicals and dead polymer molecules but they are not included herein since the polymer molecular weight distribution does not have influences on system stability and dynamics. The reactor boiling constraint is relaxed and the reactor-wall capacitance term is no longer needed due to the under-pressure operation, as this model is proposed for full-scale reactors. Table 3 shows the rate constants [7], design/operating variables at steady state and other parameters used in analysis of solution polymerization of vinyl acetate.

3. Nonlinear Dynamics in Continuous VA Polymerization

In this section we present a stationary bifurcation analysis including the continuation of steady-state solutions of the vinyl acetate polymerization system, as well as the continuation of limit points and Hopf bifurcation points. Initiator concentration in feedstream I_f , cooling water temperature T_c , reactor feed temperature T_f , residence time θ , initiator dissociation efficiency f , and volume fraction of solvent in the feed φ_s are chosen as main or second bifurcation parameters due to their ease of manipulation.

The Hopf bifurcation points and limit cycles, directly related to oscillatory behavior in solution polymerization of vinyl acetate system,

Table 3
Model parameters used in analysis of solution polymerization of vinyl acetate in a CSTR

Rate constants [7]	
For propagation/ $L \cdot mol^{-1} \cdot s^{-1}$	$k_p = k_{t0}^{1/2} 4.899 \times 10^2 \exp(-4869/(RT))$
For initiator decomposition/ s^{-1}	$k_d = 1.58 \times 10^{15} \exp(-30800/(RT))$
For chain transfer to monomer/ $L \cdot mol^{-1} \cdot s^{-1}$	$k_{trm} = 1.42 \times 10^{-2} \exp(-2700/(RT)) \times k_p$
For termination (gel effect correlation)	$k_t = k_{t0} \exp(-0.4407x_t - 6.7530x_t^2 - 0.3495x_t^3)$
Design and operating parameters	
Overall heat transfer coefficient	$UA = 12.0 \text{ cal} \cdot \text{min}^{-1} \cdot ^\circ\text{C}^{-1} [4,7]$
Heat transfer coefficient for loss to the surroundings	$(UA)_{amb} = 12.0 \text{ cal} \cdot \text{min}^{-1} \cdot ^\circ\text{C}^{-1} [4]$
Feed temperature	$T_f = 30^\circ\text{C} [6] \text{ or } 42^\circ\text{C} [4,7]$
Boiling temperature of reaction mixture	$T_b = 75^\circ\text{C} [3,4,7]$
Coolant temperature	$T_c = 45^\circ\text{C} [4,6] \text{ or } 40 - 45^\circ\text{C} [7]$
Volume fraction of solvent in the feed	$\varphi_s = 0.6 [6] \text{ or } 0.58 [7]$
Dimensionless heat transfer coefficient	$B = 1.4 [6]$
Reactor volume	$V = 500 \text{ ml} [7]$
Initiator feed concentration	$I_f = 0.038 \text{ mol} \cdot L^{-1} [7]$
Residence time	$\theta = 45 - 120 \text{ min} [7]$
Physical properties and heat capacities [4]	
Density of monomer/ $g \cdot ml^{-1}$	$\rho_m = 0.9584 - 1.3276 \times 10^{-3} T$
Density of polymer/ $g \cdot ml^{-1}$	$\rho_p = 1.211 - 8.496 \times 10^{-4} T$
Density of solvent/ $g \cdot ml^{-1}$	$\rho_s = 74.12/(60.21 + 0.116(T + 273.15))$
Heat capacity of monomer/ $\text{cal} \cdot \text{gmol}^{-1} \cdot ^\circ\text{C}^{-1}$	$C_{pm} = 0.470$
Heat capacity of polymer/ $\text{cal} \cdot \text{gmol}^{-1} \cdot ^\circ\text{C}^{-1}$	$C_{pp} = 0.3453 + 9.55 \times 10^{-4} (T - 25)$
Heat capacity of solvent/ $\text{cal} \cdot \text{gmol}^{-1} \cdot ^\circ\text{C}^{-1}$	$C_{ps} = 0.716$
Other parameters used in simulation	
Dimensionless reference temperature	
Reference residence time	$\theta_{ref} = 60 \text{ min} [6]$
Reference (ρC_p) of reaction mixture	$(\rho C_p)_{ref} = 1000 \text{ cal} \cdot L^{-1} \cdot ^\circ\text{C}^{-1} [6]$
Molecular weight of monomer	$[MW]_m = 86.09 \text{ g} \cdot \text{mol}^{-1}$
Heat of polymerization	$\Delta H_p(25^\circ\text{C}) = -21000 \text{ cal} \cdot \text{mol}^{-1} [3]$
Initiator dissociation efficiency	$f = 0.8 [4]$
Extraneous thermal capacitance factor	$\varepsilon = 0.55 [4]$

Note: 1 cal = 4.1868J, T is in K, $R = 1.9858775 \text{ cal} \cdot \text{mol}^{-1} \cdot K^{-1}$, and x_t is the monomer conversion based on total feed (including solvent).

are identified utilizing the continuation package, CL_MATCONT, developed by Dhooge *et al.* [14], as shown in Figs. 1 and 2.

Table 4 shows the representative Hopf points in this system. Briefly, Hopf bifurcation points are those with a pair of complex eigenvalues crossing the imaginary axis and always accompanied with the change of stability characteristics. Of special interest is that the unstable branch between the two Hopf bifurcation points in Fig. 2 is in a practically preferable operating region. When residence time θ is chosen as the bifurcation parameter, a Hopf bifurcation point is identified at $\theta = \theta_H = 2469.27 \text{ s}$ with the first Lyapunov coefficient $l_1 = -65.29 < 0$. This means that the system has a unique and stable limit cycle and this is a supercritical Hopf bifurcation.

Fig. 3 depicts the time series with residence time $\theta = \theta_H$ where the stable equilibrium is replaced by small-amplitude oscillations within an attracting domain. In contrast, when the coolant temperature T_c is chosen as the bifurcation parameter, a Hopf bifurcation point at $T_{cH1} = 309.245 \text{ K}$ accompanies with the positive first Lyapunov

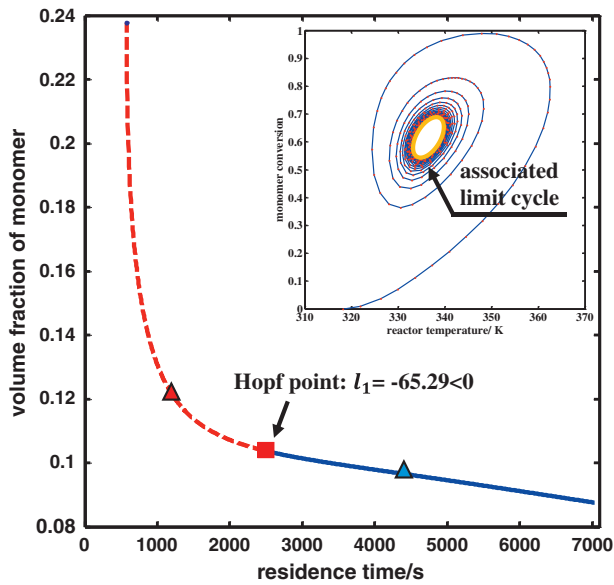


Fig. 1. Solution diagram and Hopf bifurcation points in solution polymerization of vinyl acetate in a CSTR with residence time θ as bifurcation parameter.

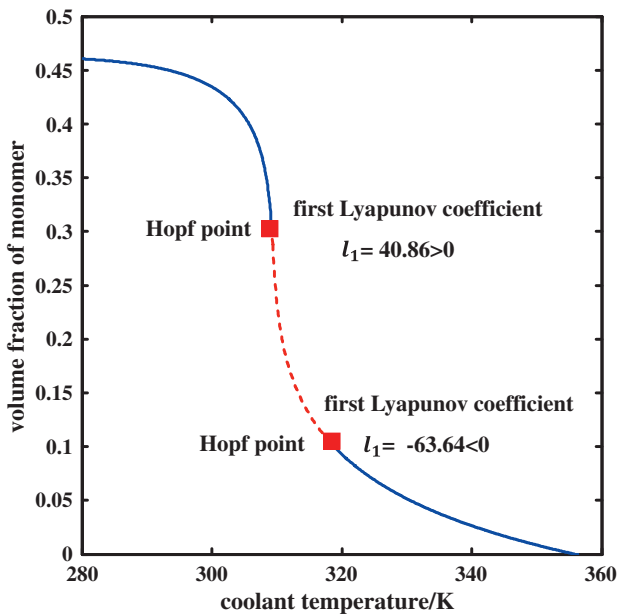


Fig. 2. Solution diagram and Hopf bifurcation points in solution polymerization of vinyl acetate in a CSTR with coolant temperature T_c as bifurcation parameter.

coefficient $l_1 = 40.86 > 0$. In this case a unique and unstable limit cycle will be generated from the subcritical Hopf bifurcation [9,15–17] in which the domain of attraction of the stable focus shrinks, which is more dangerous.

Table 4

Representative Hopf points detected in solution polymerization of vinyl acetate in a CSTR

Main bifurcation parameter		State variables			
		v_m	v_s	z_1	$y = T/T_c$
Residence time	$\theta_H = 2469.27$ s	0.103758	0.593366	0.964456	1.05606
Coolant temperature	$T_{cH1} = 309.245$ K	0.298020	0.623502	1.05250	1.05352
Coolant temperature	$T_{cH2} = 318.319$ K	0.103018	0.593240	0.963265	1.05638

Fig. 4 shows the dynamics at the left triangle point on the unstable dashed branch in Fig. 1, while Fig. 5 depicts the time series at the right triangle point on the blue solid stable branch in Fig. 1 where no periodic solution exists and the operating point is asymptotically stable.

4. Bifurcation Modification Based on Eigenvalue Assignment

Herein we adopt the washout filter based on the technique of eigenvalue assignment to achieve the purpose of bifurcation control.

The necessity of implementing bifurcation control instead of traditional state feedback or/and adaptive control is closely related to the nature of solution polymerization of vinyl acetate in a CSTR. Fig. 6 shows the bifurcation diagram and Hopf bifurcation points with initiator dissociation efficiency f as the main bifurcation parameter and residence time θ as the second one. All the detected Hopf bifurcation points and limit points, which indicate the system stability changes, form the dot line–“Hopf curve” and further split the parameter space into unstable region A and stable region B. For a practical polymerization process, the initiator dissociation efficiency f generally ranges between 0.5 and 0.8, and the most practical preferable operating regions are medium to high reactor temperatures with higher monomer conversion. Unfortunately, this industrially preferable operating region is largely overlapped with unstable branches inside region A in Fig. 6.

On the other hand, Fig. 7 depicts corresponding bifurcation diagrams with initiator concentration I_f as the main bifurcation parameter and residence time θ as the second one. As the residence time increases, the unstable branch shrinks. However, the extremely long residence time up to 7200 s is not preferable from the practical point of view. Thus the key problems turn to whether it would be possible to operate under relatively short residence time on the dashed unstable branches with higher monomer conversions. The main benefit of using washout filters is that all the equilibrium points of the open-loop system are preserved [18]. Thus we can concentrate on the design of controllers emphasizing the improvement in performance for a particular operating point in region A in Fig. 6 or dashed unstable branches in Fig. 7.

We begin with considering the original continuous-time nonlinear system described by

$$\dot{\mathbf{x}} = \mathbf{F}^\mu(\mathbf{x}, \mathbf{u}) \quad (11)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^l$ is the input vector, μ is the bifurcation parameter, and vector field $\mathbf{F}: \mathbb{R}^n \times \mathbb{R}^l \rightarrow \mathbb{R}^n$ is a nonlinear function of \mathbf{x} and \mathbf{u} , which is assumed to be sufficiently smooth. We expands $\mathbf{F}(\cdot, \cdot)$ about the operating point of interest, say \mathbf{x}_e , yielding

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}_e, \mathbf{u}_e)\mathbf{x} + \mathbf{B}(\mathbf{x}_e, \mathbf{u}_e)\mathbf{u} + h.o.t.(\text{higher order term}) \quad (12)$$

where \mathbf{A} is the Jacobian matrix of the open-loop system and \mathbf{A} has element $A_{i,j}$ on its i th row and j th column

$$A_{i,j} = \frac{\partial F_i(\mathbf{x}, \mathbf{u})}{\partial x_j}, \quad i, j = 1, 2, \dots, n \quad (13)$$

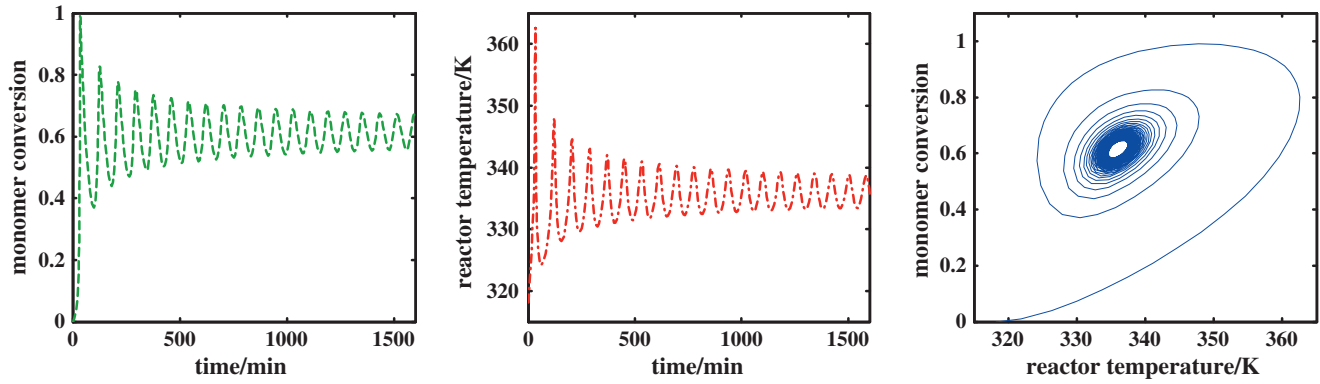


Fig. 3. Time series with residence time $\theta = \theta_H$ and corresponding limit cycle on the phase panel, $I_f = 0.038 \text{ mol} \cdot \text{L}^{-1}$, $\varphi_s = 0.58$.

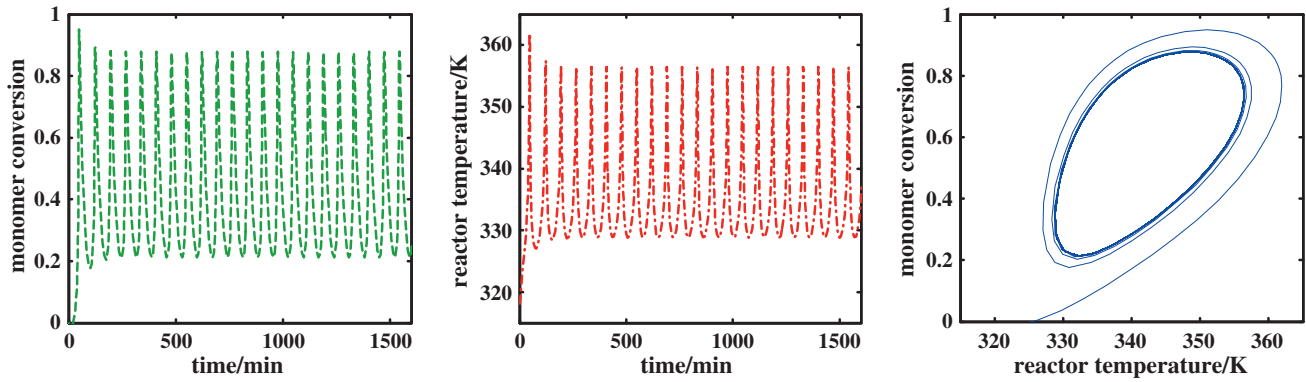


Fig. 4. Time series with residence time $\theta < \theta_H$ and corresponding limit cycle on phase panel, $I_f = 0.038 \text{ mol} \cdot \text{L}^{-1}$, $\varphi_s = 0.58$.

where \mathbf{B} has element B_{iq} on its i th row and q th column

$$B_{i,q} = \frac{\partial F_i(\mathbf{x}, \mathbf{u})}{\partial u_q}, \quad i = 1, 2, \dots, n, \text{ and } q = 1, 2, \dots, l. \quad (14)$$

Washout filter-aided feedback can be viewed as a special form of output feedback where the output of the washout filters instead of the open-loop system states is used in the feedback.

The transfer function of this introduced washout filter is

$$G(s) = \frac{y_k(s)}{x_k(s)} = \frac{s}{s+d} = 1 - \frac{d}{s+d}. \quad (15)$$

The washout filter is governed by the dynamic equation

$$\dot{z} = x_k - dz \quad (16)$$

along with the output equation and controller expressed as

$$y_k = x_k - dz \quad (17)$$

$$\mathbf{u} = \mathbf{u}(y) = \mathbf{v} + \mathbf{w}y_k + h.o.t. \quad (18)$$

Herein, d is the reciprocal of the filter time constant, which is positive for a stable filter and negative for an unstable filter.

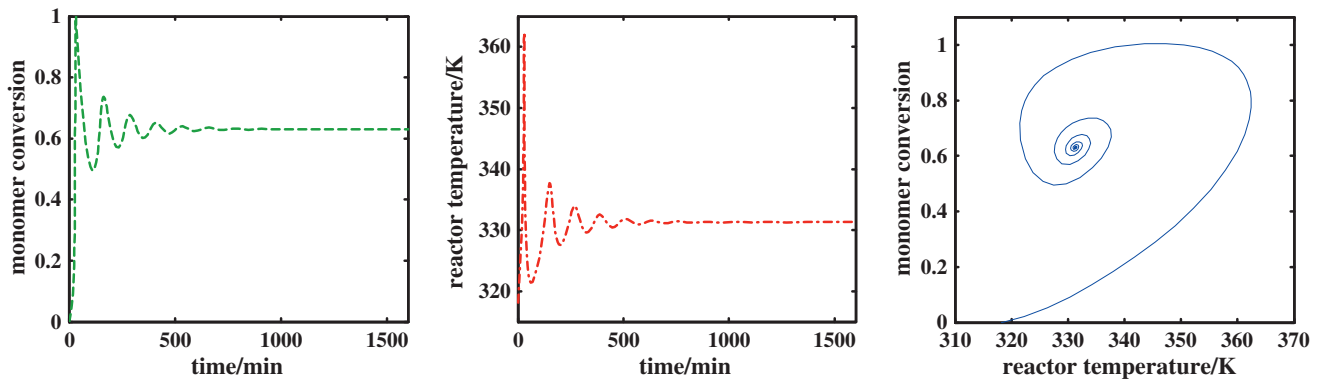


Fig. 5. Time series with residence time $\theta > \theta_H$ where no limit cycle exists, $I_f = 0.038 \text{ mol} \cdot \text{L}^{-1}$, $\varphi_s = 0.58$.

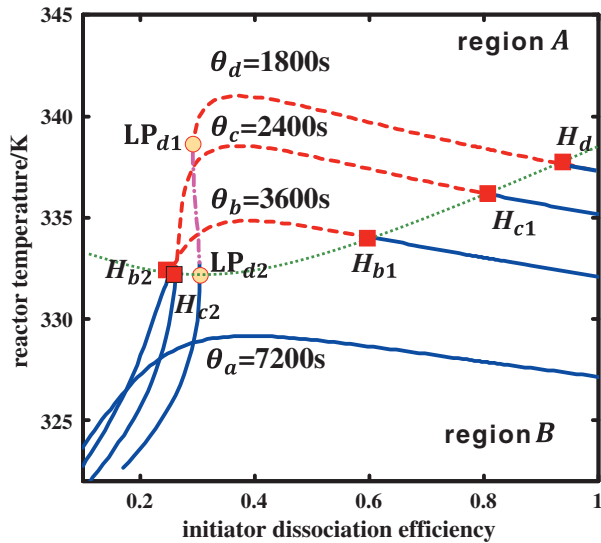


Fig. 6. Bifurcation diagram and Hopf bifurcation points with initiator dissociation efficiency f as main bifurcation parameter and residence time θ as the second one.

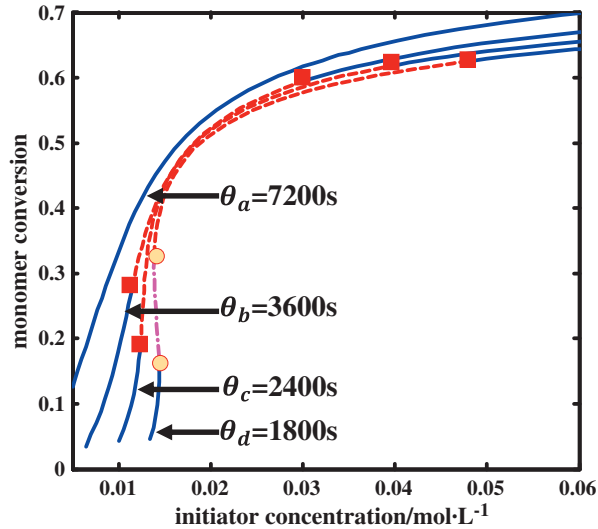


Fig. 7. Bifurcation diagram and Hopf bifurcation points with initiator concentration I_f as the main bifurcation parameter and residence time θ as the second one.

Fig. 8 compares the natural or typical procedures of implementing bifurcation control and washout filters. The procedure on the left has been used for determining the transcritical, pitchfork, and saddle-nodde types of bifurcations, as well as stabilities of equilibria in Lorenz system [9]. Although controllers could be designed by following this typical procedure, tremendous computational effort has to be made for bifurcation modification purpose. Instead of the traditional bifurcation control, the technique of washout filters is based on bifurcations closely related to the eigenvalues of linearized model. By eigenstructure assignment we mean to control the behavior of eigenvalues in an appropriate way and use washout filters to alter the eigenvalues and/or eigenvectors of the VA polymerization system. This new procedure has two very desirable properties. First, the expression of control law is very simple. For engineering applications, it might be more preferable to have a simplest possible design. Second, the control law is solved consciously for a given purpose of bifurcation control. No iteration loop is needed. Besides, only the first four steps are essential steps and the rest four steps only exist for verification purposes. Note that it is

necessary to identify the freedom available in partial eigenstructure assignment and select linearly independent vectors to ensure that the matrix of eigenvectors is as well-conditioned as possible.

Fig. 9 gives the original bifurcation diagram with the preferable steady state operating point $(\mathbf{x}_e, \mathbf{u}_e)$ on the unstable branch with high monomer conversion and mild but sufficiently high reactor temperature.

$$\mathbf{x}_e = [v_{me}, v_{se}, z_{ie}, y_{Te}] = [0.28932, 0.62144, 1.0281, 1.0348] \quad (19)$$

In this part, initiator dissociation efficiency f and volume fraction of solvent in feed φ_s are fixed to be 0.8 and 0.58, respectively, and are not included in the input vector since they are not so straightforward to observe. The rest of the bifurcation parameters are selected as the input vector

$$\mathbf{u}_e = [I_0, T_{jc}, T_{f0}, \theta] = [0.038, 318.15, 303.23, 1800]. \quad (20)$$

The linearization of the original system described in Eqs. (1)–(4) is given as

$$\mathbf{A} = \begin{bmatrix} -2.8665 & -0.01822 & -0.00020728 & -8.7681 \times 10^{-5} \\ 0 & 2.9571 \times 10^{-5} & 5.7732 \times 10^{-5} & 0 \\ 0 & -0.0026598 & 0.0012993 & -5.6321 \times 10^{-6} \\ 1.7147 & 0.0091425 & 0.0031922 & 4.5783 \times 10^{-5} \end{bmatrix} \quad (21)$$

$$\mathbf{B} = \begin{bmatrix} -2.8665 & -0.01822 & -0.00020728 & -8.7681 \times 10^{-5} \\ 0 & 2.9571 \times 10^{-5} & 5.7732 \times 10^{-5} & 0 \\ 0 & -0.0026598 & 0.0012993 & -5.6321 \times 10^{-6} \\ 1.7147 & 0.0091425 & 0.0031922 & 4.5783 \times 10^{-5} \end{bmatrix} \quad (22)$$

The original four eigenvalues of the Jacobian matrix at $(\mathbf{x}_e, \mathbf{u}_e)$ are

$$\text{original set of eigenvalues} = \begin{bmatrix} 0.56661 \\ -0.21444 \\ -0.95397 + 0.011574i \\ -0.95397 - 0.011574i \end{bmatrix} \quad (23)$$

After implementing the bifurcation control using the technique of washout filters, the single positive eigenvalue is successfully eliminated by two arbitrarily given eigenvalues -0.1 and -0.2 . And the four dimensional system is augmented to five dimensional one.

$$\text{new set of eigenvalues} = \begin{bmatrix} -0.10000 \\ -0.20000 \\ -0.21444 \\ -0.95397 + 0.011574i \\ -0.95397 - 0.011574i \end{bmatrix} \quad (24)$$

with corresponding controller parameters given as

$$\mathbf{w} = \begin{bmatrix} -189.67 \\ -13400 \\ 11998 \\ 8.923 \times 10^6 \end{bmatrix} \quad \mathbf{d} = -0.035298. \quad (25)$$

Fig. 10 gives the bifurcation diagram after implementing bifurcation control using washout filters. The unstable solution branch where the targeted operating point was located has been rendered to stable branch since the two newly introduced eigenvalues are artificially placed in the open right half of the complex plane. All the desired operating points and the shape of the original equilibrium of the open-loop system are preserved.

Various k values in the output Eq. (17) are tested and corresponding controller parameters are listed in Table 5. It is found that the filter time

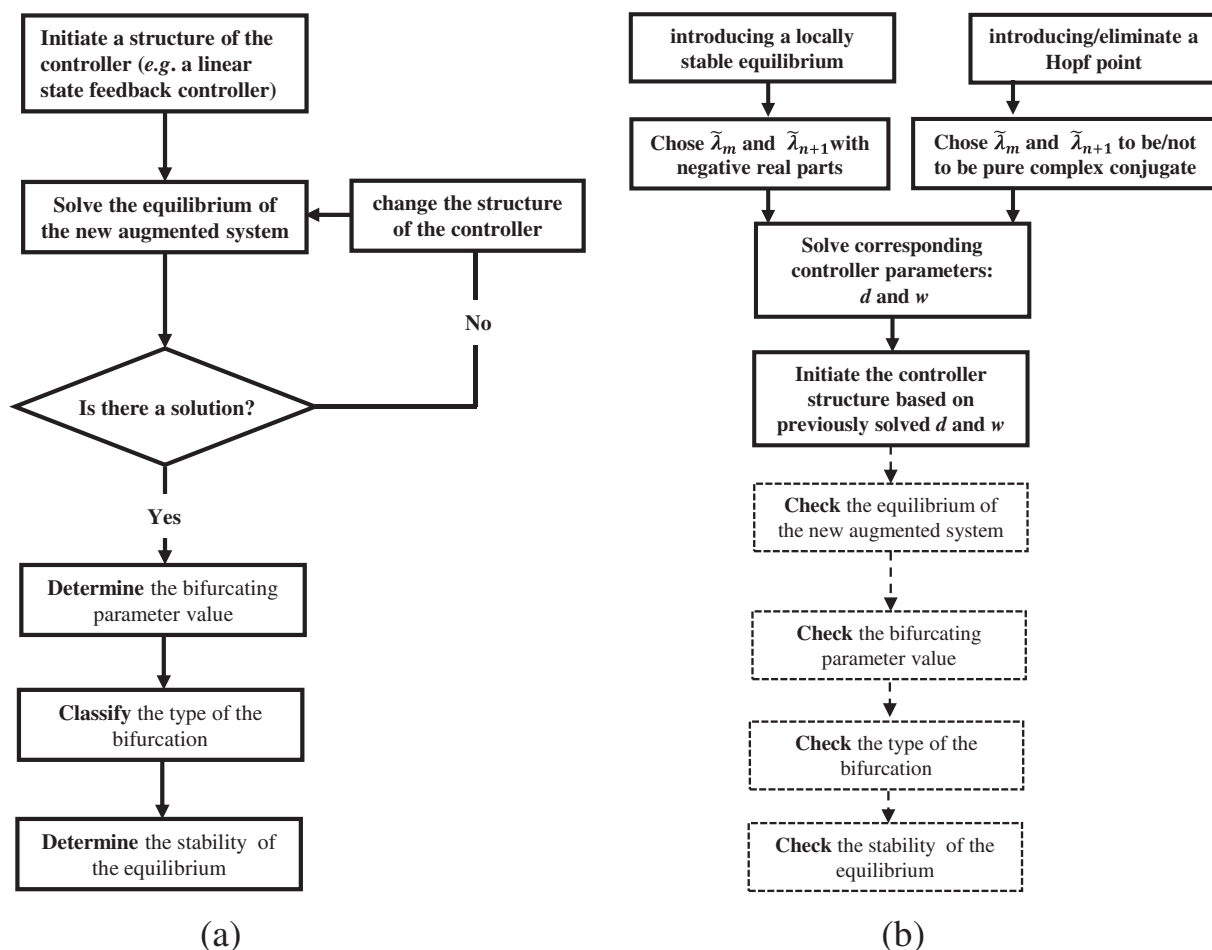


Fig. 8. Comparison between a natural (or typical) procedure of implementing bifurcation control (a) and the new procedure using washout filters to implement bifurcation control (b).

constant d in the washout filter will not be influenced by the selection of k , and is determined only by the assigned eigenvalues.

5. Conclusions

This paper discussed the nonlinear behavior of the parameterized nonlinear system of continuous solution polymerization of vinyl acetate

(VA). Limit points such as Hopf bifurcation point corresponding to periodic solutions are obtained. A new procedure of implementing bifurcation control is applied to modify the bifurcation characteristics of VA polymerization process via a washout filter-aid controller. Obtained control law successfully preserves the practically preferable equilibria of the VA polymerization, and this is of much greater importance when the procedure is applied in future models with both

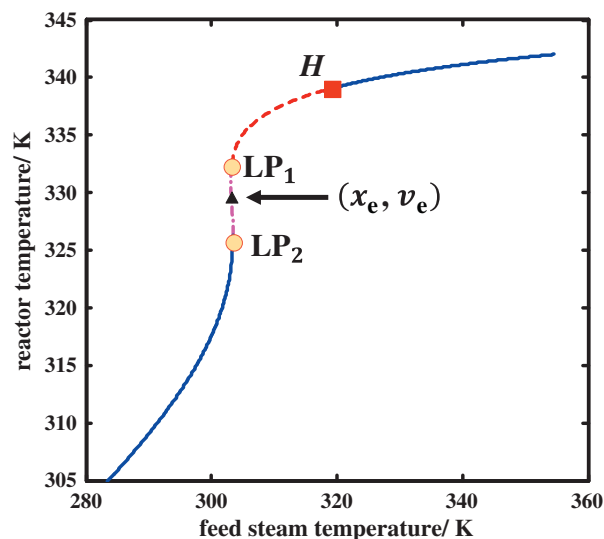


Fig. 9. Original bifurcation diagram with preferable operating point.

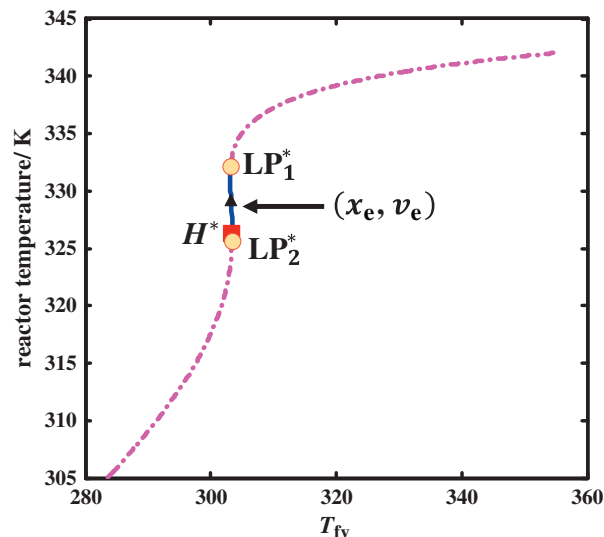


Fig. 10. Bifurcation diagram after implementing bifurcation control using washout filter.

Table 5

Eigenvalue assignment of solution polymerization of vinyl acetate in a CSTR

	Original eigenvalue	Assigned eigenvalues		Controller parameters				
	$\lambda_m = \lambda_1$	$\tilde{\lambda}_m$	$\tilde{\lambda}_{m+1}$	w_1	w_2	w_3	w_4	d
$k = 1$	0.56661	−1.0	−2.0	452.19	31947	−28605	-2.1273×10^7	−3.5298
$k = 2$	0.56661	−1.0	−2.0	4540.5	3.2078×10^5	-2.8723×10^5	-2.1361×10^8	−3.5298
$k = 3$	0.56661	−1.0	−2.0	1380.1	97506	−87307	-6.4929×10^7	−3.5298
$k = 4$	0.56661	−1.0	−2.0	−1492.4	-1.0543×10^5	94406	7.0208×10^7	−3.5298
$k = 4$	0.56661	−0.1	−0.2	−189.67	−13400	11998	8.923×10^6	−0.035298

parameter/nonparameter (structural) uncertainties where the operating point is not precisely known in advance or adequate information of system model is not available.

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